Deep Inelastic Scattering

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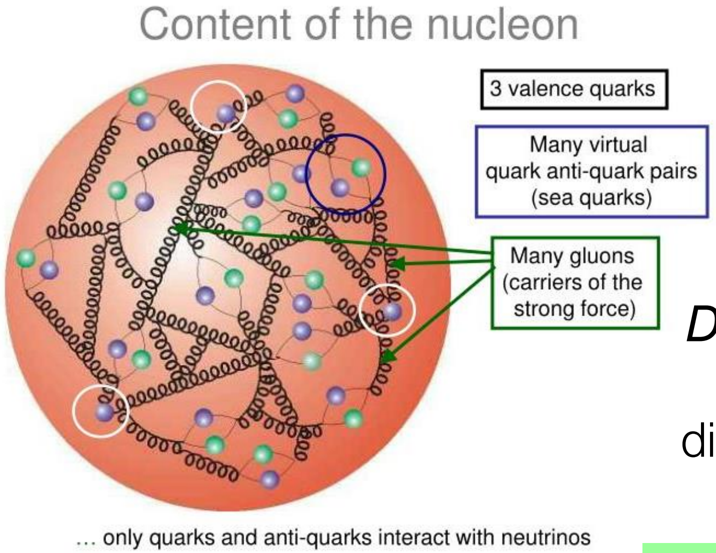
Electron – Proton scattering

The proton has a very complex structure.
How to study it?

- How deep the virtual particle can penetrate the proton depends on the equivalent wavelength of the exchanged virtual photon:

$$\lambda \ll R \quad \rightarrow \quad Q^2 \gg \hbar^2/R^2$$

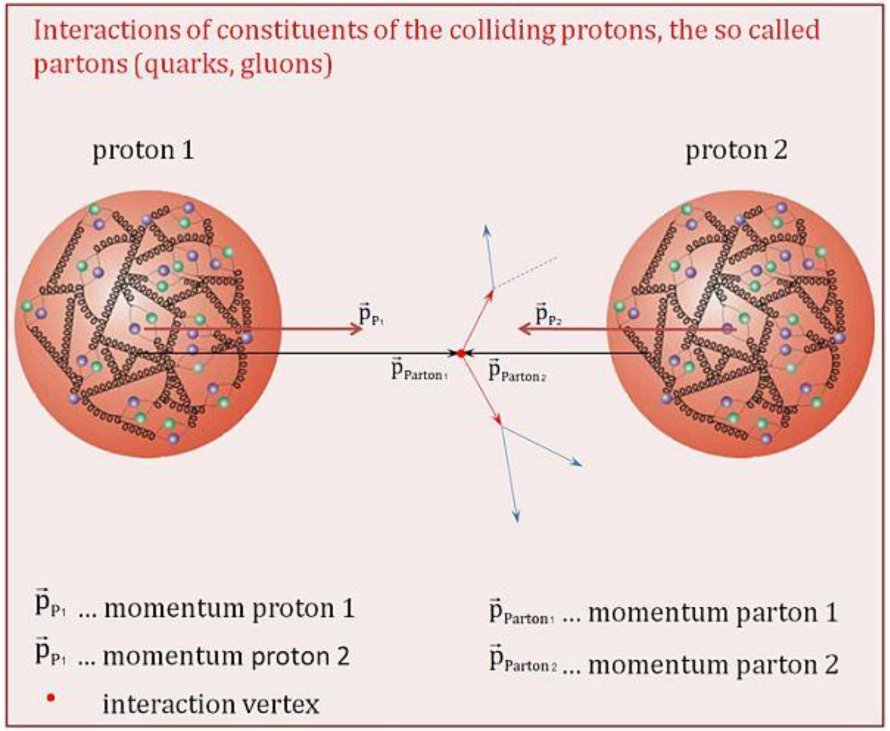


- At low energies, ***elastic scattering***, the proton remains intact. Interaction between a photon and a proton (as a whole) → global properties of the proton such as radius.
- At high energies, ***deep inelastic scattering***, the proton breaks up. Is interpreted as the elastic scattering of the electron from one of the quarks within the proton.

Deep inelastic scattering gives the momentum distribution of the quarks.

$e^- p$ best way to study proton

Proton – proton scattering



Why Deep Inelastic Scattering?

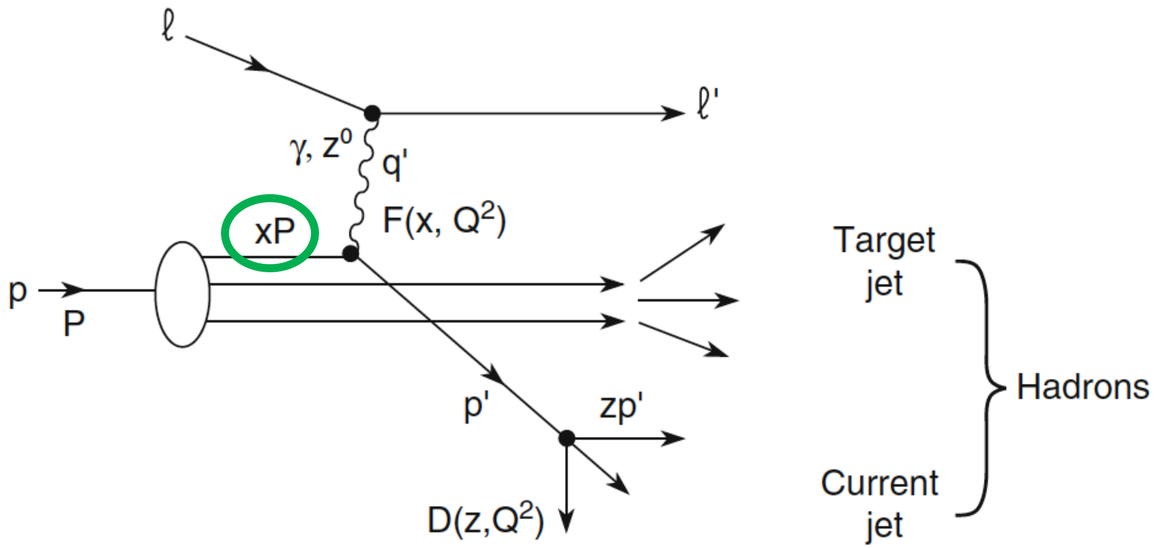
This course is titled "Experimental High Energy Physics at Colliders"

- Colliders are today the most powerful instrument to study the innermost structure of matter
- Proton-proton colliders are the accelerators that can reach the highest energies, for reasons that will be clear when discussing about accelerators
- Proton are very complex objects, with a complex internal structure
- The interpretation of scattering experiments need to be based on the understanding of the proton structure
- The scattering lepton-nucleon allows us to study the structure of the proton

Deep Inelastic Scattering → *DIS*

Deep Inelastic Scattering → ***USE leptons***

Basis of QCD, the theory of hadronic interactions



Many generation of scattering experiments.

- Initially they used leptons (mostly electrons) produced in accelerators and sent on a target
- The last generation was the HERA collider at Desy, Germany

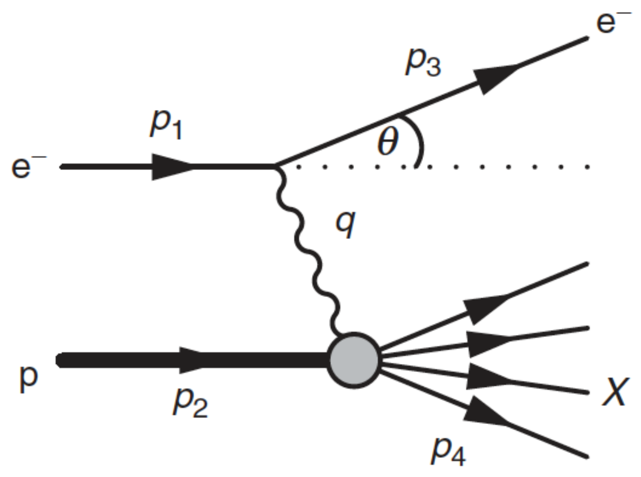
30 GeV electrons against 900 GeV protons

Deep Inelastic Scattering (DIS)

- DIS =
  - *QED* interaction of a virtual photon with the constituent quarks inside the proton (electrons & muons);
  - *Weak* interactions induced by a ν can also give information on the structure of hadrons;

- A lot of experimental data, the measured structure functions describe the momentum distributions of the quarks.

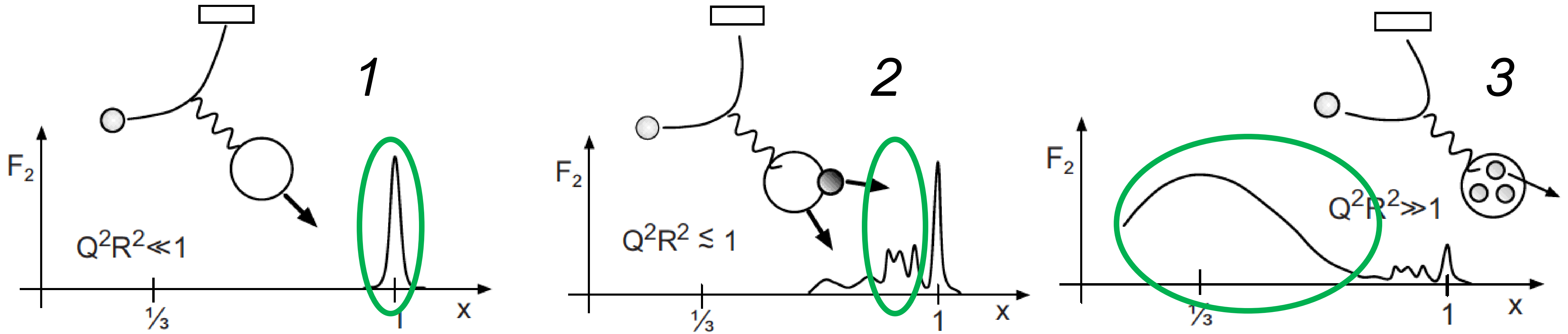
- The proton is found to be a complex dynamical system made of quarks, gluons and antiquarks.



DIS measurements interpreted using x. This variable is generally known as "Bjorken scaling variable" and gives an indication of the inelasticity of the process.

- $x = 1 \rightarrow elastic\ scattering;$
- $x < 1 \rightarrow inelastic\ scattering\ on\ costituents\ of\ the\ proton$

Start to understand 'x'



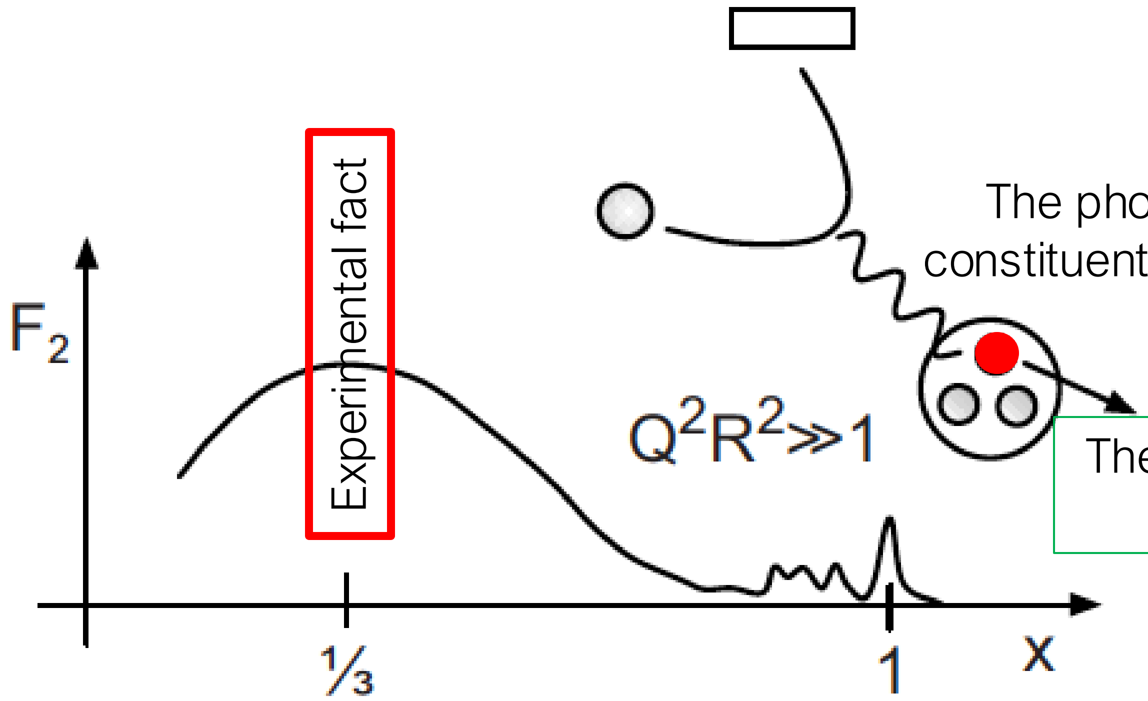
What do we see with increasing Q^2?

See above 3 different cases

Q2 ↑ wave length of the probe particle ↓

1. The Q^2 of the reaction is ~low, the **nucleon** is seen by the exchanged photon as **a unique obj**ect. We have elastic scattering
2. The Q^2 of the reaction is not as ~low as in 1, not enough to probe the inner structure but enough to **excite the nucleon**
3. The Q^2 of the reaction is ~large enough to see the internal structure of the proton and the photon scatters elastically on one of the **internal constituents of the nucleon**

More Understanding of ‘x’



The photon hits one constituent of the nucleon

The constituent of the nucleon is scattered elastically

The constituent of the quark cannot be observed → it ‘transforms’ into several hadrons (”it hadronizes”) (will see this later)

The peak at ~1/3 can be understood as the “most probable” x value corresponding to the *elastic scattering of the photon and one of the nucleon constituents.*

If we assume that the ‘x’ budget is equally shared by ‘**n**’ nucleon constituents then

$$\boldsymbol{x = \frac{1}{n}\frac{Q^2}{2Pq} = \frac{1}{n}\frac{Q^2}{2M\nu}}$$

This term is equal to 1 in case of elastic scattering

$$\frac{1}{3} = \frac{1}{n}$$ → there are 3 components in the nucleon

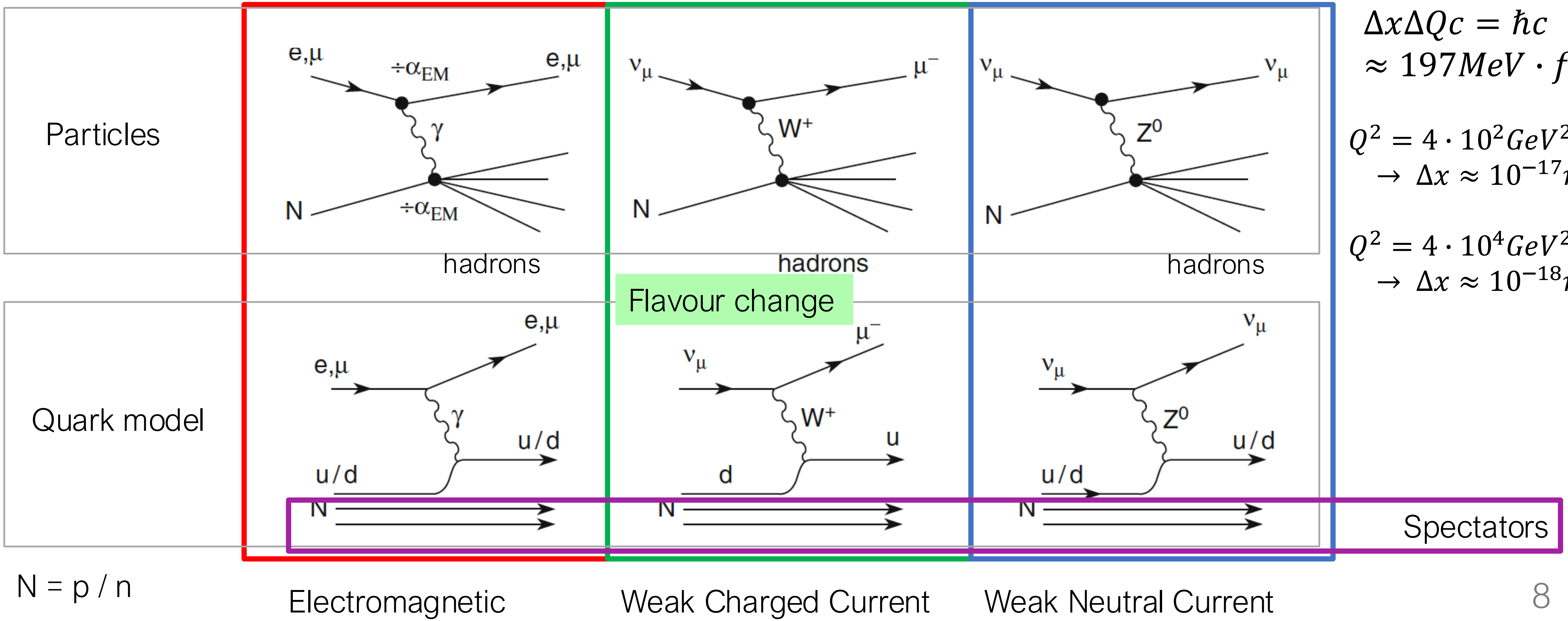
Inelastic Lepton-Nucleus Scattering

$ep: \quad e^{\pm} + p \to e^{\pm} + X^{+}$

$\mu p: \quad \mu^{\pm} + p \to \mu^{\pm} + X^{+}$

$\nu_\mu p(CC): \nu_\mu + p \to \mu^- + X^{++}, \quad \overline{\nu}_\mu + p \to \mu^+ + X^0$

$\nu_\mu p(NC): \quad \nu_\mu + p \to \nu_\mu + X^{+}, \quad \overline{\nu}_\mu + p \to \overline{\nu}_\mu + X^{+}.$



$\Delta x \Delta Q c = \hbar c$
$\approx 197 MeV \cdot fm$

$Q^2 = 4 \cdot 10^2 GeV^2/c^2$
$\to \Delta x \approx 10^{-17} m$

$Q^2 = 4 \cdot 10^4 GeV^2/c^2$
$\to \Delta x \approx 10^{-18} m$

Electron – Proton scattering: History

Studying the nucleon's constituents the wave length of the probe particle λ has to be small compared to the nucleon's radius, R

$$\lambda \ll R \;\rightarrow\; Q^2 \gg \hbar^2/R^2$$

Large Q^2 values are needed → high energies are required.

- The first generation ~1960 @ **SLAC** 25 GeV electrons on a target
- The second generation ~ 1980 @ **CERN** using beams of *muons* of up to 300 GeV (*).

Beams on target

- The last generation ~1990 → 2007 @ **DESY** Collider **HERA:** 30 GeV electrons against 900 GeV protons (see next slides).

Collider

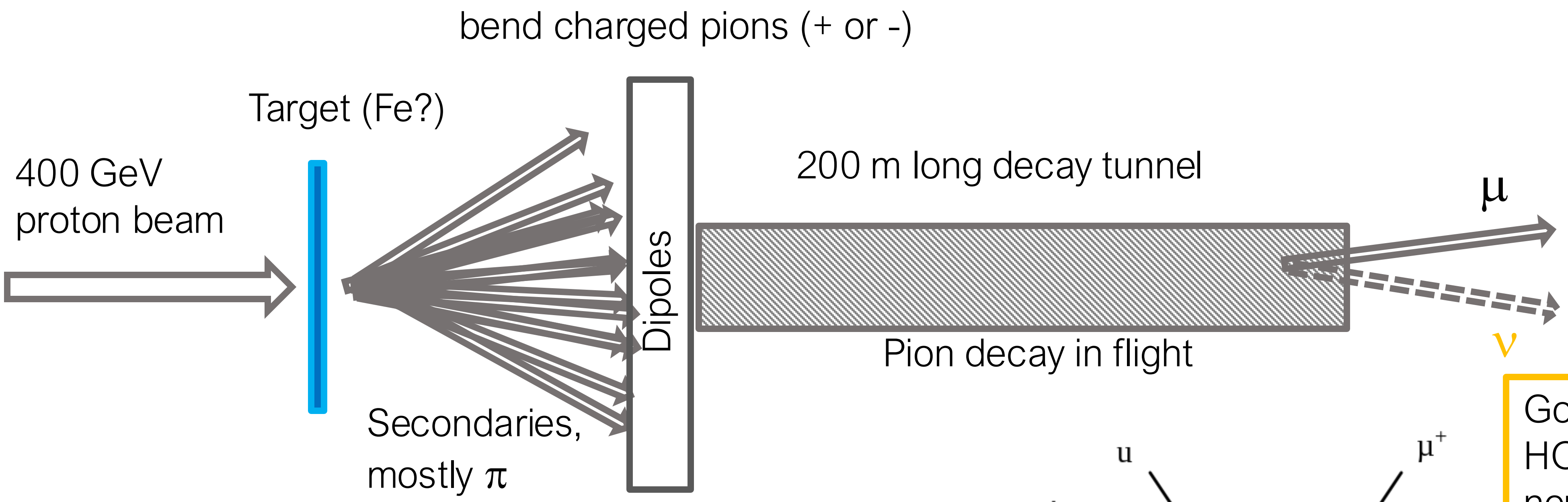
- ➢ In the SLAC experiments, the basic properties of the quark and gluon structure of the hadrons were established.
- ➢ The second and the third generations of experiments are at the basis of the

Quantum Chromodynamics,

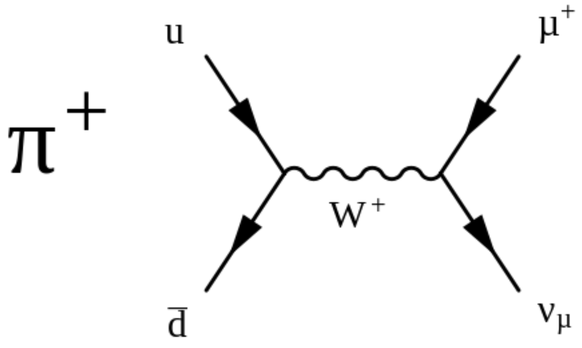
the theory of the strong interaction.

(*) Protons of 400 GeV on a target produced pions which were kept confined in a 200 meters tunnel. During the flight part of the pions decayed into muons which were collected into a beam with energies up to 300 GeV.

Producing Muon Beams



Goes ~undetected HOWEVER may give a neutrino beam



$\pi^0 \rightarrow \gamma + \gamma$

p + Fe → X (~1/3 π^+ + 1/3 π^- + 1/3 π^0) + ~ p (1/2 energy)

$\pi^- \rightarrow \mu^- + \bar{\nu}$

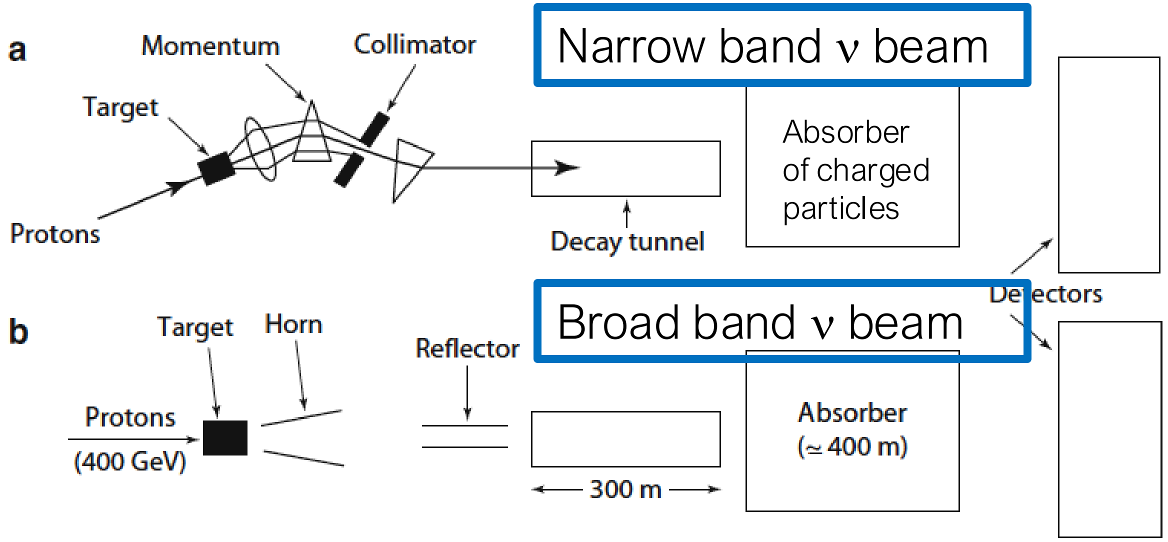
$\pi^+ \rightarrow \mu^+ + \nu$

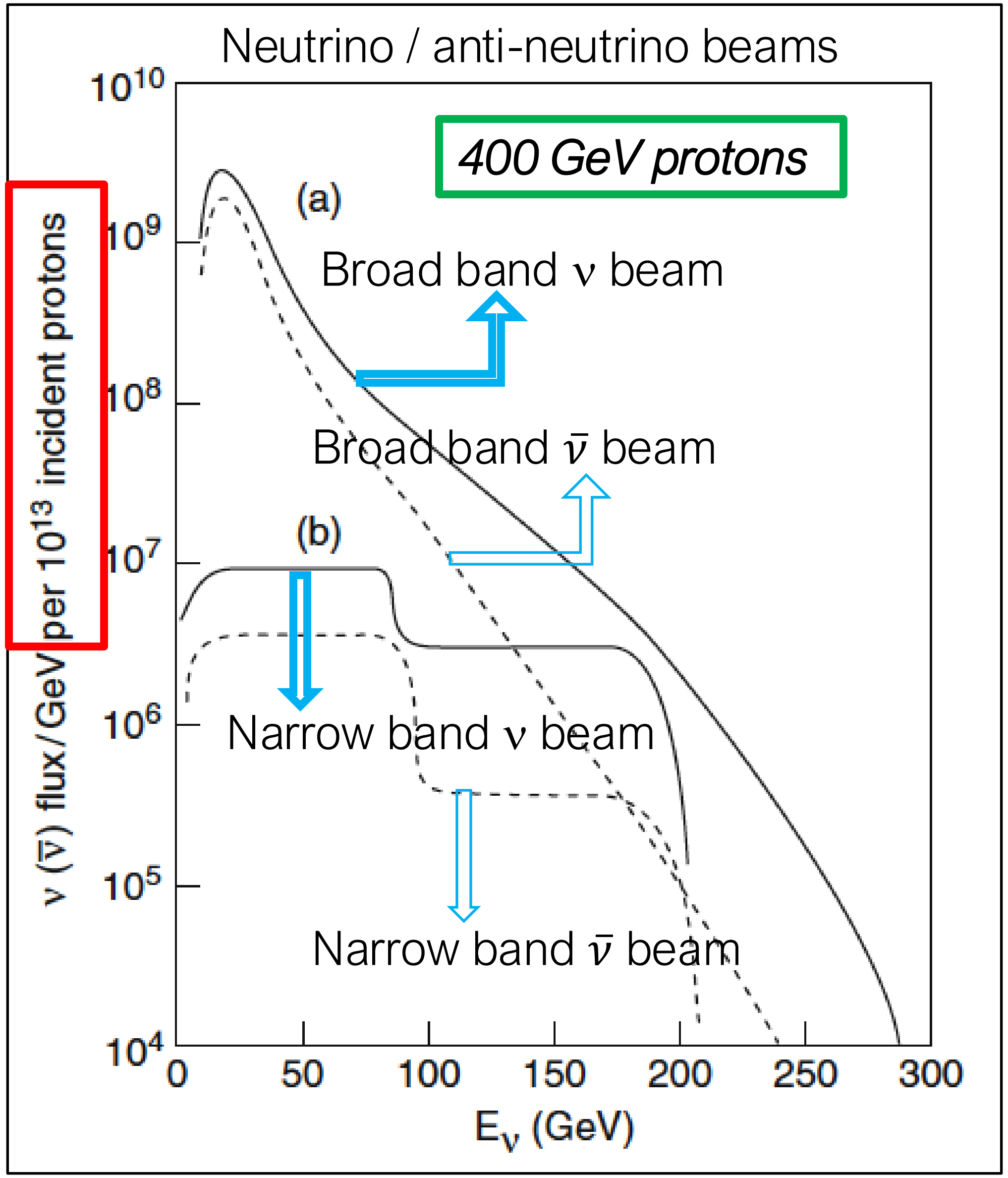
π^+ *gives* ν *and* μ^+

π^- *gives* $\bar{\nu}$ *and* μ^-

Producing Neutrino Beams

$\pi^- \to \mu^- + \bar{\nu}$
$\pi^+ \to \mu^+ + \nu$





Narrow band ν beam: ~ selected in momentum ~ low intensity
Broad band ν beam: ~ not selected in momentum ~ high intensity

Experiments:

The mean free path in iron of 10GeV neutrinos is $\lambda \approx 2.6 \cdot 10^9 Km$ (~ 20 cm for hadrons!). This means that only a very small fraction $3 \cdot 10^{-13}$ of 10 GeV neutrinos interact in a meter of iron. With a flux of 10^{12} neutrinos (for 10^{13} accelerated protons incident on the target), there are only 0.3 interactions in one meter of iron.

→ very long and massive detectors

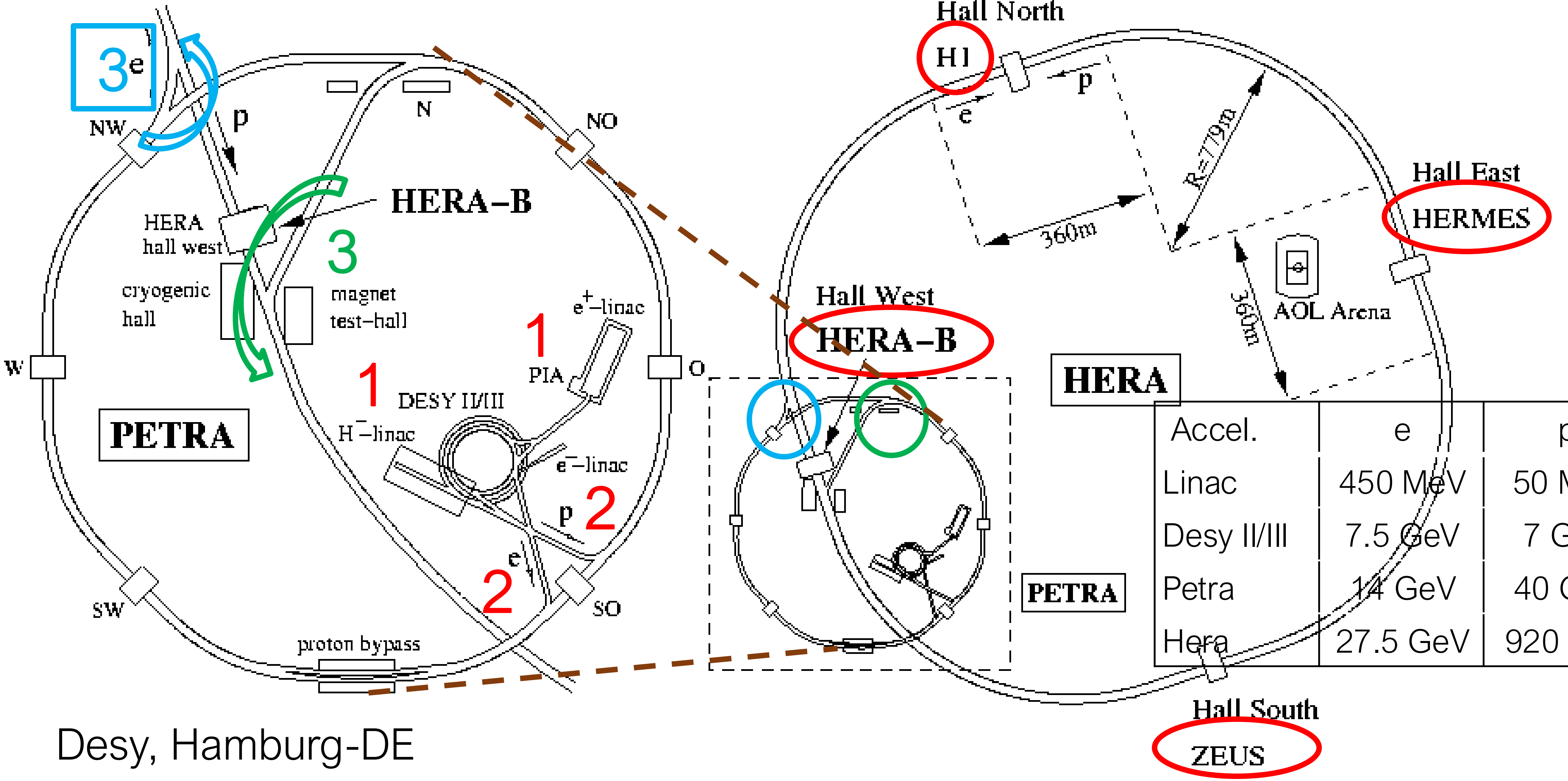
Hera, Hadron-Electron Ring in Desy-DE

Circular $e + p$ accelerator @ Desy, Hamburg-DE.

- 15 to 30 m underground and circumference of 6.3 km. Leptons and protons → two independent rings
- At HERA, 27.5 GeV electrons (or positrons) collided with 920 GeV protons, cms energy of 318 GeV (*).
- electrons or positrons: 450 MeV, 7.5 GeV, 14 GeV, 27.5 GeV.
- Protons: 50 MeV, 7 GeV, 40 GeV, 920 GeV.
- 4 interaction regions, 4 experiments H1, ZEUS, HERMES and Hera-B.
- About 40 minutes to fill the machine
- Operated between 1992 and 2007.

$$(*)\ E_{cm}(or\ cms) = \sqrt{m_p^2 + m_e^2 + 2E_pE_e(1 - \beta_1\beta_2\cos(\theta))} \approx \sqrt{2E_pE_e \cdot 2}$$

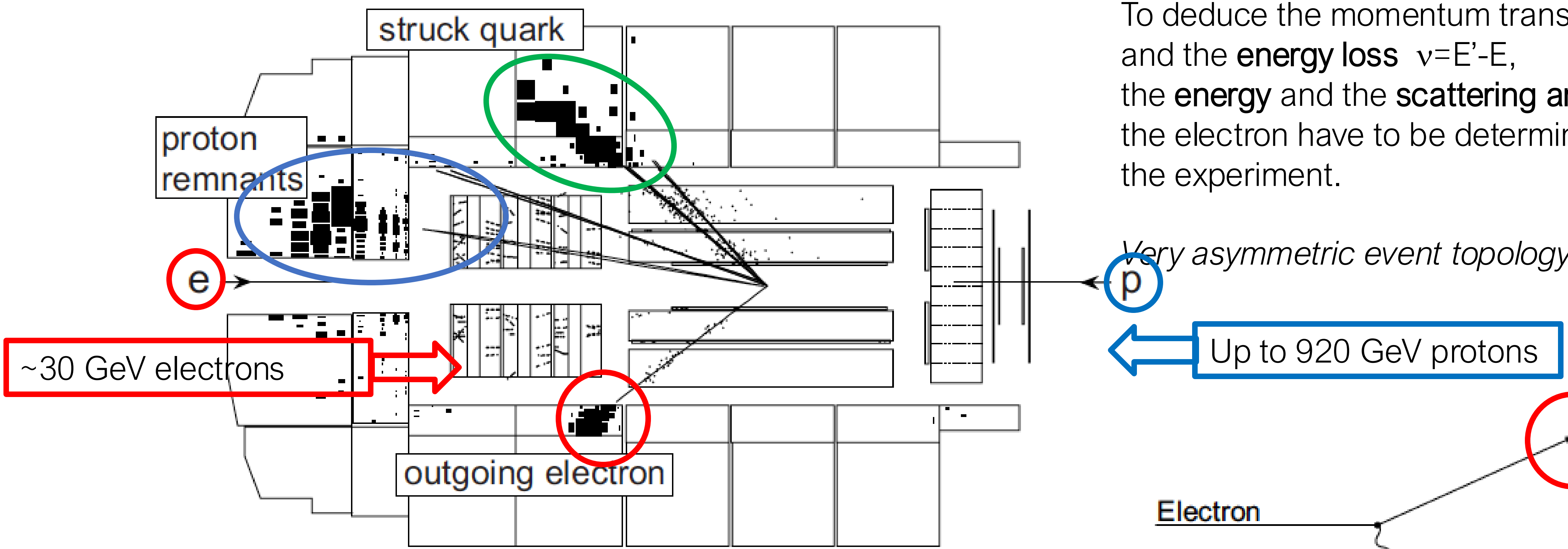
Three stage acceleration: Linac and *Petra and then Hera*



Accel.	e	p
Linac	450 MeV	50 MeV
Desy II/III	7.5 GeV	7 GeV
Petra	14 GeV	40 GeV
Hera	27.5 GeV	920 GeV

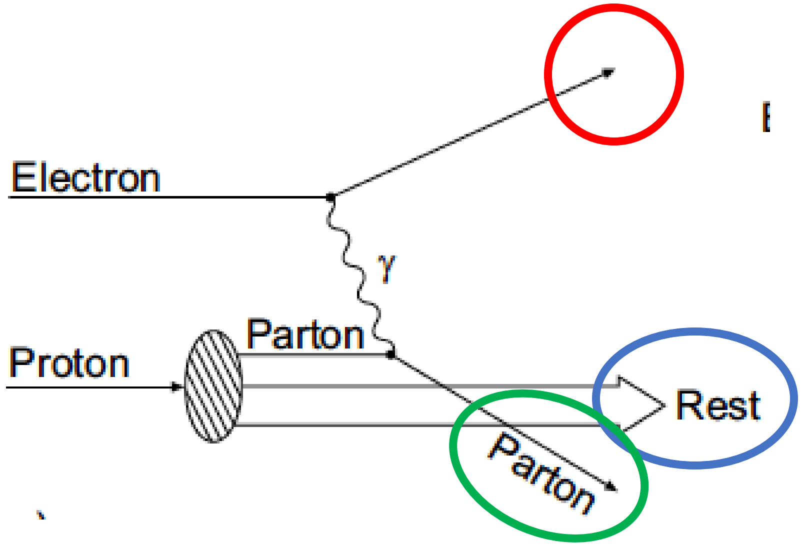
Desy, Hamburg-DE

Display of one DIS event in Hera



To deduce the momentum transfer Q^2 and the **energy loss** ν=E'-E, the **energy** and the **scattering angle** of the electron have to be determined in the experiment.

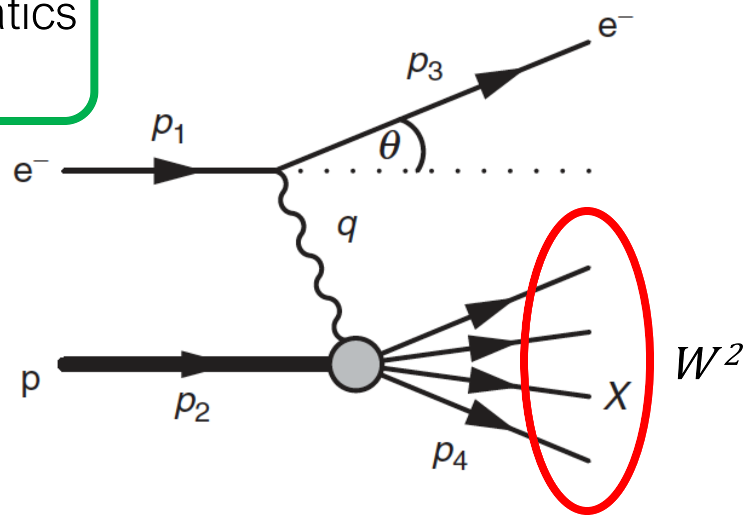
Very asymmetric event topology!



The direction of all charged particles is measured in the inner tracking detector. The energy of the scattered electron is measured in the electromagnetic calorimeter, that of the hadrons in the hadron calorimeter.

DIS Variables

- Elastic Scattering e against point-like proton: 1 variable only determines kinematics
- Inelastic Scattering electron against *complex* proton: 2 variables are needed.



Possible choices:

- Q^2 negative of the 4-momentum of the virtual photon $\boldsymbol{Q^2 = -q^2}$;

$$Q^2 = -(p_1 - p_3)^2 = -2m_e^2 + 2p_1 \cdot p_3 = -2m_e^2 + 2E_1E_3 - 2\mathrm{p}_1\mathrm{p}_3 \cos\theta.$$

At high Q^2 the electron mass can be neglected and $p \sim E$

$$Q^2 \approx 2E_1E_3(1 - \cos\theta) = 4E_1E_3 \sin^2\frac{\theta}{2},$$

- Bjorken $\boldsymbol{x = \frac{Q^2}{2p_2 \cdot q}}$ Get physical meaning by computing the invariant mass of the hadronic system W^2

$$W^2 \equiv p_4^2 = (q + p_2)^2 = q^2 + 2p_2 \cdot q + p_2^2 \qquad x = \frac{Q^2}{Q^2 + W^2 - m_p^2}$$

Elasticity of the interaction

$$\Rightarrow \quad W^2 + Q^2 - m_p^2 = 2p_2 \cdot q,$$

If $\boldsymbol{x = 1} \rightarrow W^2 = m_p^2 \rightarrow$ elastic scattering

$$0 \le x \le 1.$$

You always have one baryon (p lightest) $W^2 \equiv p_4^2 \ge m_p^2$

dimensionless

DIS Variables

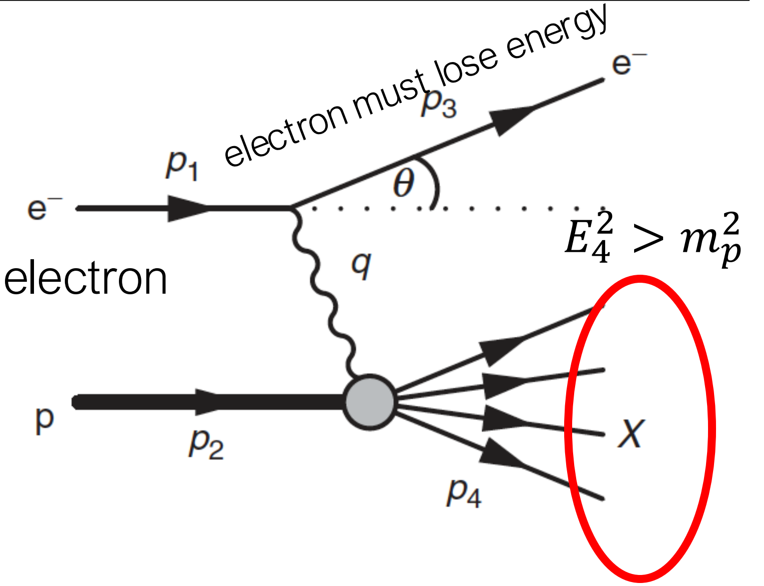
The inelasticity y

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}.$$

dimensionless

In the frame where the proton is at rest

$p_1 = (E_1, 0,0, E_1)$ incoming electron
$p_2 = (m_p, 0,0,0)$ proton at rest
$p_3 = (E_3, E_3 \cdot \sin(\theta), 0, E_3 \cdot \cos(\theta))$ outgoing electron
$q = (E_1 - E_3, \boldsymbol{p_1} - \boldsymbol{p_3})$



$$y = \frac{m_p(E_1 - E_3)}{m_p E_1} = 1 - \frac{E_3}{E_1}$$

Relative energy lost by the e (%)

$$0 \leq y \leq 1.$$

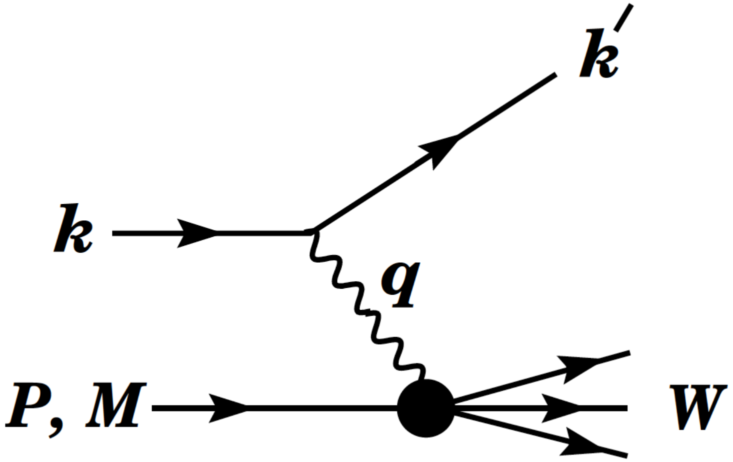
The electron energy loss ν

$$\nu \equiv \frac{p_2 \cdot q}{m_p},$$

In the system where the proton is at rest ν is the energy lost by the incoming electron

$$\nu = E_1 - E_3,$$

Summary of DIS Invariant Quantities



- E, E' *initial and final lepton energy*
- θ *lepton scattering angle*
- M *nucleon mass*

$\nu = \frac{q \cdot P}{M} = E - E'$	*lepton's energy loss*
$Q^2 = -q^2$ $= 2(EE' - \vec{k} \cdot \vec{k'}) - m_\ell^2 - m_{\ell'}^2$	Q^2 *value*
$if: EE' \sin^2(\frac{\theta}{2}) \gg m_\ell^2, m_{\ell'}^2$, *then* $Q^2 \approx 4\, EE' \sin^2(\frac{\theta}{2})$	Q^2 *value when* $m_\ell^2, m_{\ell'}^2$ *negligeable*
$x = \frac{Q^2}{2M\nu}$	*fraction of the nucleon's momentum carried by the struck quark*
$y = \frac{q \cdot P}{k \cdot P} = \frac{\nu}{E}$	*fraction of the lepton's energy lost in the nucleon rest frame*
$W^2 = (P + q)^2 = M^2 + 2M\nu\text{-}Q^2$	*mass squared of the system recoiling against the scattered lepton*
$s = (k + P)^2 = \frac{Q^2}{xy} + M^2 + m_\ell^2$	*lepton-nucleon center-of-mass energy*

Elastic scattering: kinematics determined by
- ➢ θ *lepton scattering angle*

Inelastic scattering: kinematics determined by
- ➢ θ *lepton scattering angle*
- ➢ E' *final lepton energy*

DIS at ~Low Q^2

$p_1 = (E_1, 0,0, E_1)$ incoming electron
$p_2 = (m_p, 0,0,0)$ proton at rest
$p_3 = (E_3, E_3 \cdot \sin(\theta), 0, E_3 \cdot \cos(\theta))$
$q = (E_1 - E_3, \boldsymbol{p_1} - \boldsymbol{p_3})$

At ~low Q^2 both elastic and inelastic scattering can happen

2 variables needed to describe kinematics $\rightarrow \frac{d^2\sigma}{d\Omega dE} \rightarrow E, \Omega\ (\rightarrow \theta)$: Final electron energy & electron scattering angle

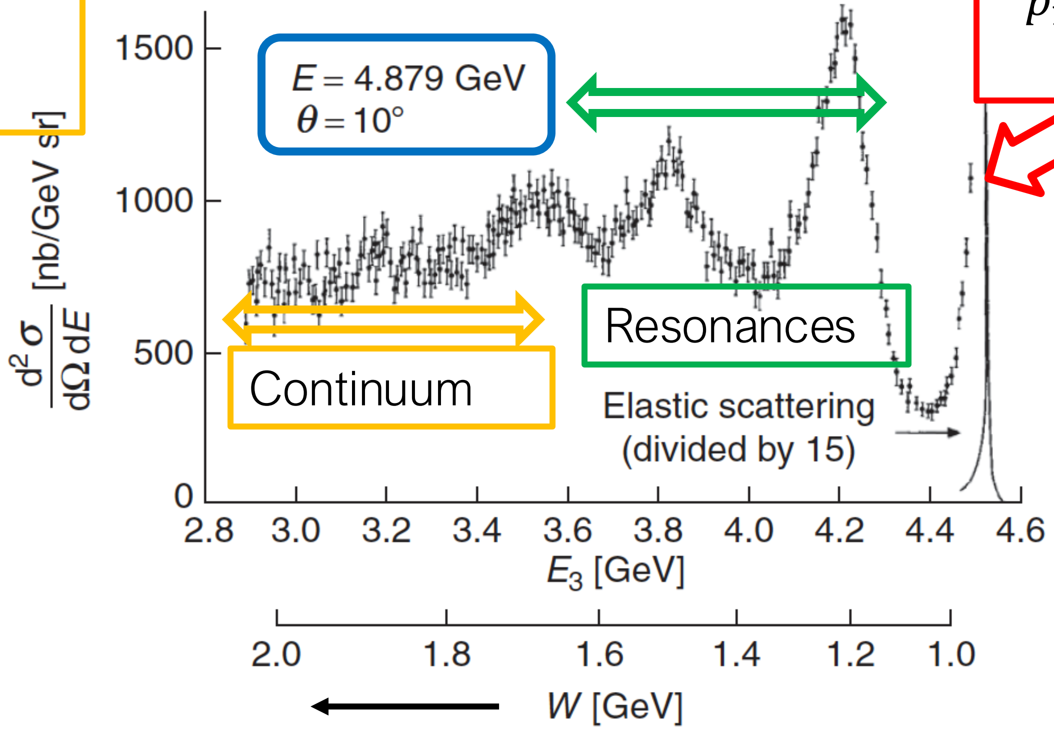
$$W^2 = (p_2 + q)^2 = p_2^2 + 2p_2 \cdot q + q^2 = m_p^2 + 2p_2 \cdot (p_1 - p_3) + (p_1 - p_3)^2$$

$$\approx \left[m_p^2 + 2m_p E_1\right] - 2\left[m_p + E_1(1 - \cos\theta)\right] E_3.$$

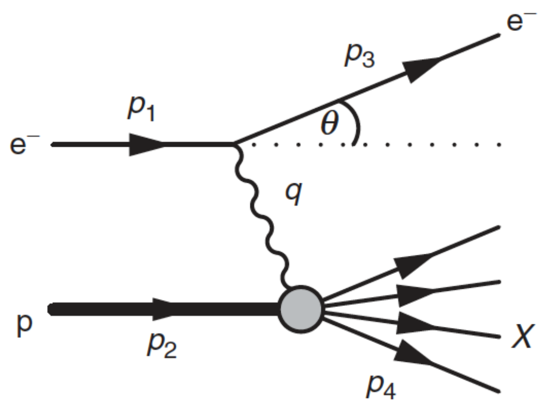
The continuum: start of DIS → the proton is broken → multi-particle final states.

resonances = excited bound states of the proton (quarks composition uud), which subsequently decay strongly.
Width Γ, lifetime τ

$$\Gamma = \frac{1}{\tau}$$



$p_1 = 4.879, \theta = 10°, E_3 \approx 4.5\ GeV \rightarrow m_p$
$\rightarrow$ *elastic scattering*



Low Q^2 DIS measurements

Rosenbluth Formula

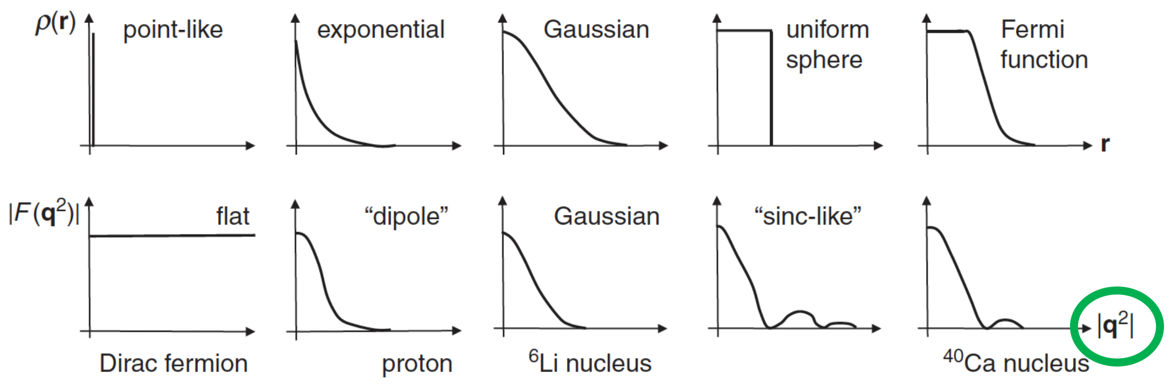
$$\frac{d\sigma}{d\Omega} = \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} + 2\tau G_M^2 \tan^2\frac{\theta}{2}\right)\left(\frac{d\sigma}{d\Omega}\right)_0$$

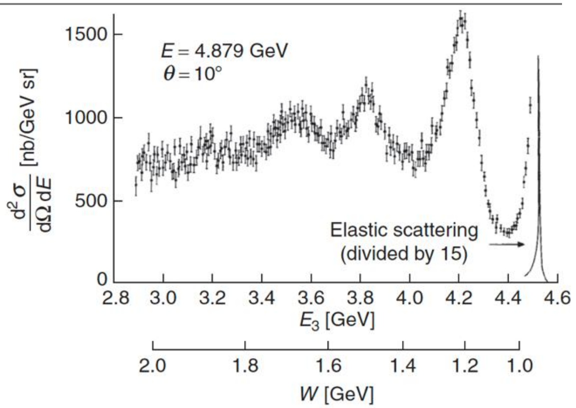
$$\tau = \frac{Q^2}{M^2c^2}$$

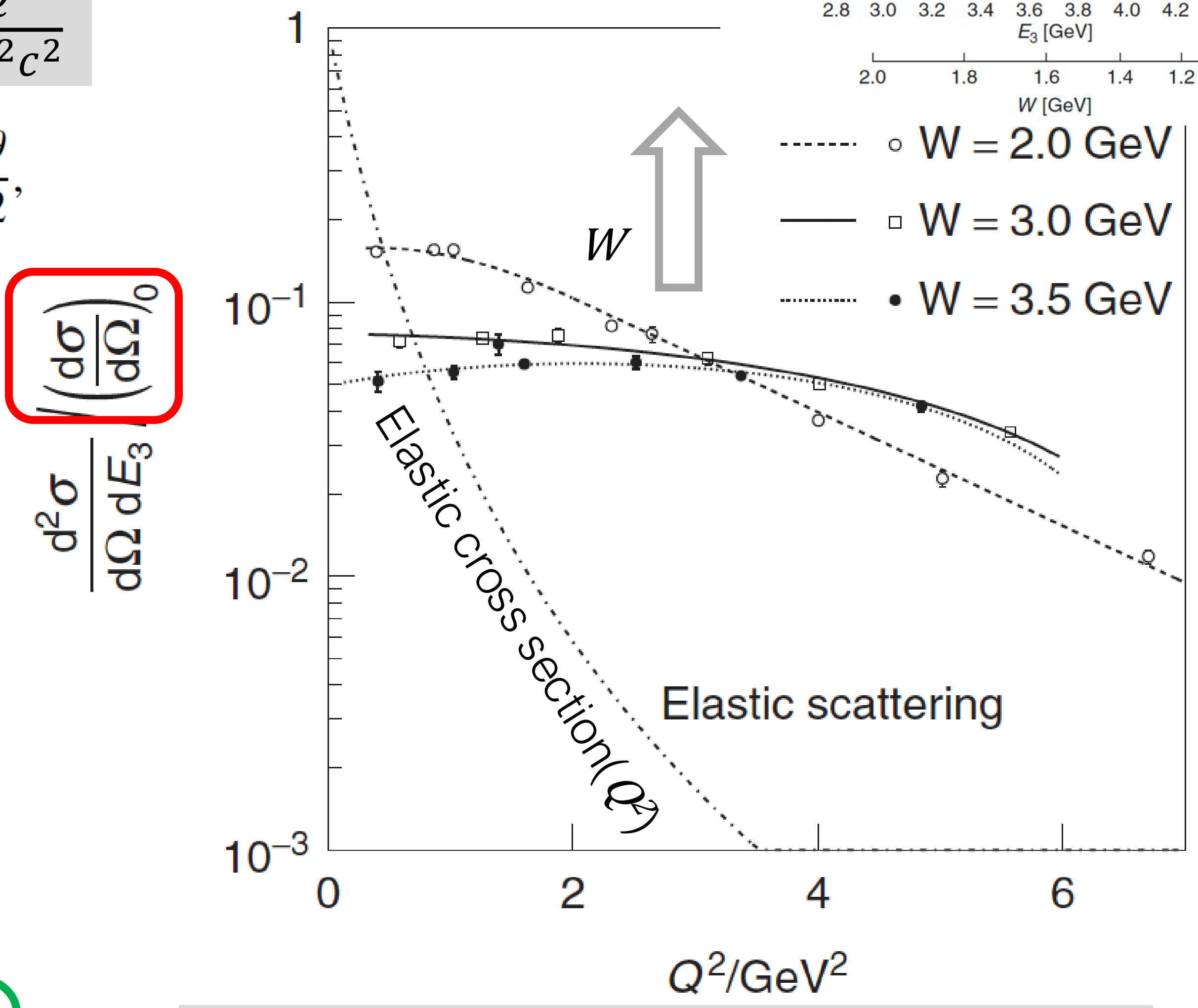
Mott cross section (point-like proton)

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{\alpha^2}{4E_1^2\sin^4(\theta/2)}\left(\frac{E_3}{E_1}\right)\cos^2\frac{\theta}{2},$$

- At high values of W (hadronic system) → dependence on Q^2 much weaker than for elastic scattering;
- The higher the mass (→ more in DIS region) the smaller the dependence;
- Qualitatively explained by Form Factor $\frac{F_{Dirac}(Q^2)}{F_{Proton}(Q^2)}$







Low Q2 → low resolution of the lepton (probe) → lepton sees all the proton charge

$p_1 = (E_1, 0,0, E_1)$ incoming electron
$p_2 = (m_p, 0,0,0)$ proton at rest
$p_3 = (E_3, E_3 \cdot \sin(\theta), 0, E_3 \cdot \cos(\theta))$
$q = (E_1 - E_3, \boldsymbol{p_1} - \boldsymbol{p_3})$

Rosenbluth formula is the most general expression for the *elastic* scattering cross section

$$ep \to ep$$

Puts into evidence electric and magnetic form factors $G_E, G_M = functions(q^2)$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4(\theta/2)} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1+\tau)} \cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2} \right)$$

$$\tau = \frac{Q^2}{4m_p^2}$$

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1}.$$

Can be rewritten (in Lorentz-invariant form) by

- introducing two new functions: $f_1(Q^2)$, magnetic only, and $f_2(Q^2)$, magnetic and electric;
- Using y (for elastic scattering x=1 and y depends only on Q^2)

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

$$x = \frac{Q^2}{2p_2 \cdot q}$$

$$1 = \frac{Q^2}{2p_2 \cdot q} \to 2p_2 \cdot q = Q^2$$

$$\to y = Q^2/(2 \cdot p_2 \cdot p_1)$$

$p_2 \cdot p_1$ constant

Elastic scattering, proton doesn't break, $x=1 \to y$ depends on Q^2 only

Structure Functions: Inelastic Scattering

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2)\right]$$

Elastic scattering 1 variable $f(Q^2)$

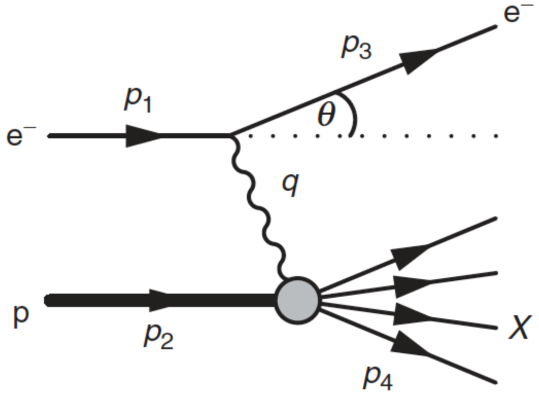
$$\frac{d^2\sigma}{dx\, dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[\left(1 - y - \frac{m_p^2 y^2}{Q^2}\right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

Inelastic scattering 2 variables $F(x, Q^2)$

most general Lorentz-invariant expression for ep → eX inelastic scattering, mediated by the exchange of a single virtual photon

$F_2(x, Q^2)$ Electric & magnetic term

$F_1(x, Q^2)$ Purely magnetic term



If $m_p^2 y^2 / Q^2 \ll 1$

$$\frac{d^2\sigma}{dx\, dQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1 - y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right].$$

In a fixed target experiment $e^- p$ the values of Q^2, x *and* y can be determined by the measurement of the energy and direction of the scattered electron, E_3, θ

$$Q^2 = 4E_1 E_3 \sin^2\frac{\theta}{2}, \quad x = \frac{Q^2}{2m_p(E_1 - E_3)} \quad \text{and} \quad y = 1 - \frac{E_3}{E_1}$$

The double differential cross section $\frac{d^2\sigma}{dx dQ^2}$ can me constructed by counting how many events you have between

$$x \to x + \Delta x,\ Q^2 \to Q^2 + \Delta Q^2$$

Structure Functions & Scaling Violations

Experimental finding: $F_1(x, Q_2)$ and $F_2(x, Q_2)$ are (almost) independent of Q2

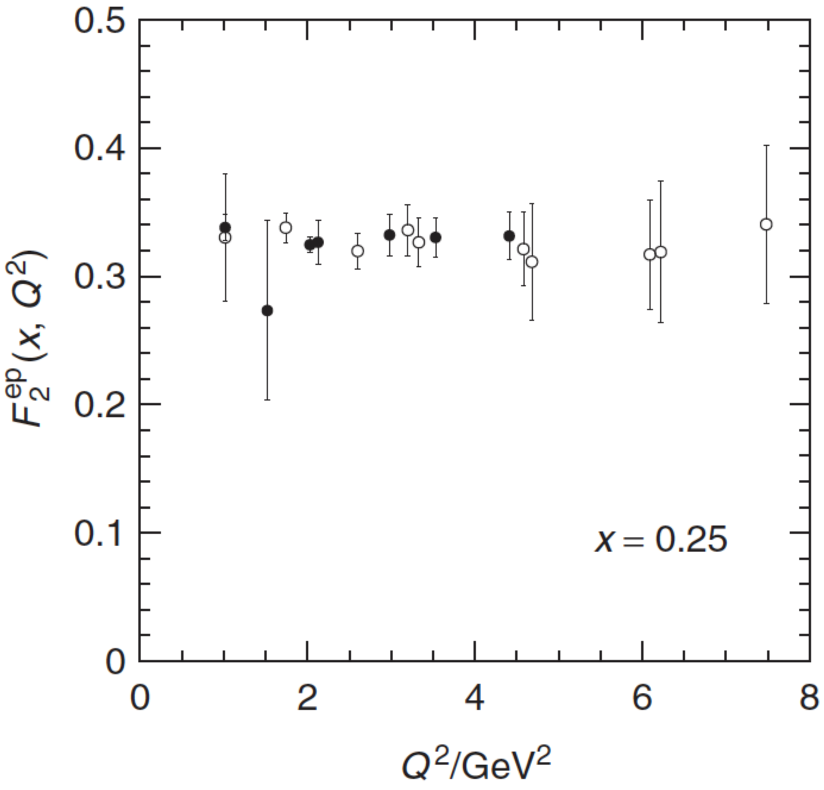
$$F_1(x, Q^2) \to F_1(x) \quad \text{and} \quad F_2(x, Q^2) \to F_2(x).$$

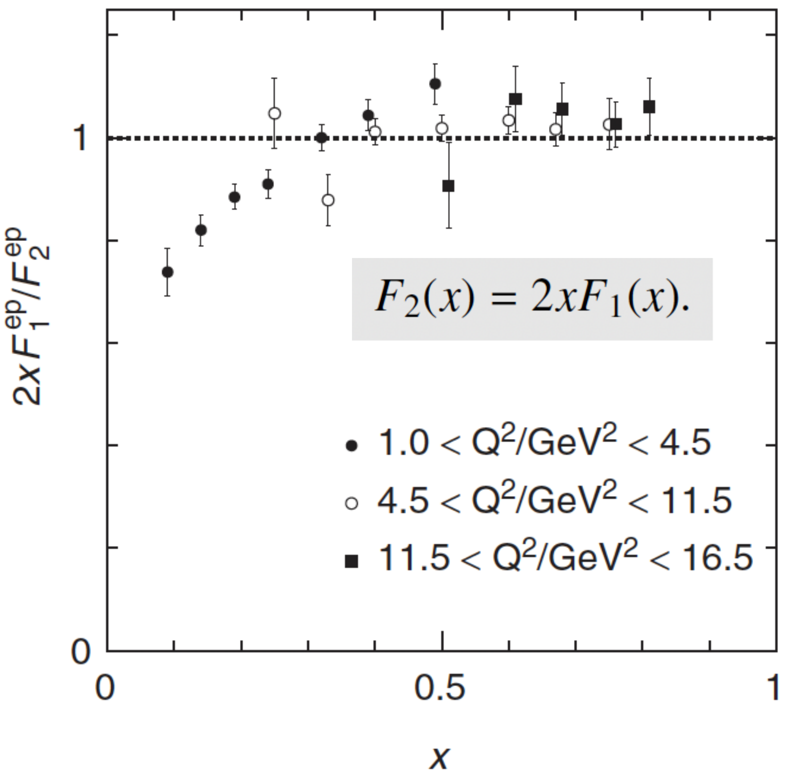
Implies: ep *inelastic* scattering = *Σelastic* scattering(s) of eq pointlike spin 1/2 constituent of the p. *Electric and magnetic contributions/components are defined by the Dirac equation*

F_1 and F_2 in inelastic scattering
$ep \to e + X$

- Experiment at the Stanford Linear Accelerator Centre (SLAC) in California.
- e^- between 5 GeV and 20 GeV on a hydrogen target.
- The scattering angle and energy of the electron was measured using a large movable spectrometer.

Point-like scattering object?



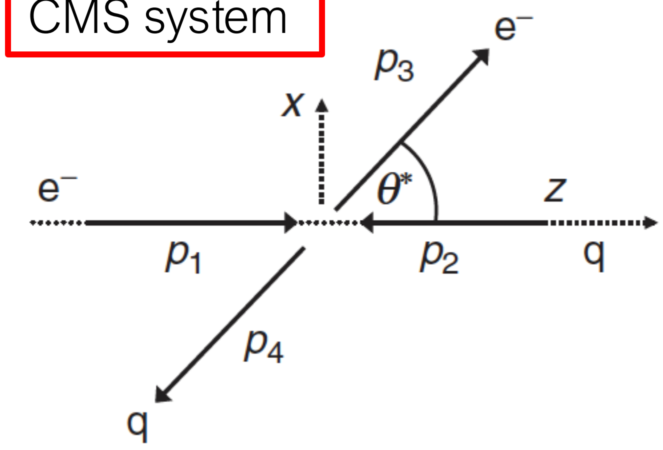


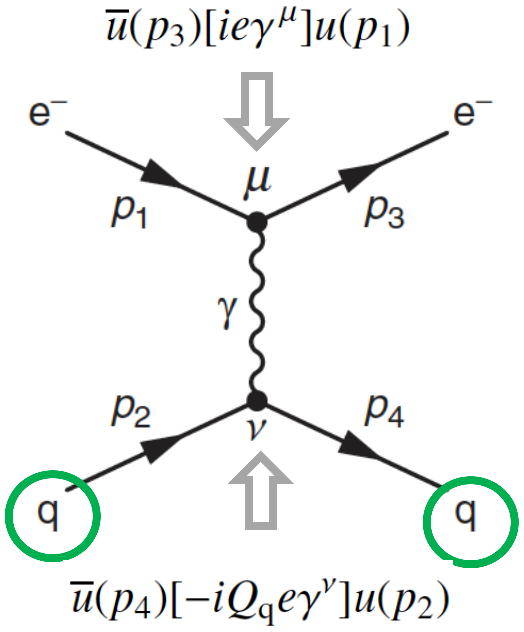
$ep \rightarrow eq$ elastic scattering

Compute Matrix element using the technique (Fermi golden rule) you can use for computing $\mathcal{M}$ in

- $e^+e^- \rightarrow \mu^+\mu^-$
- $e^-p \rightarrow e^-p$ (Rutherford & Mott & Rosenbluth formulas)

CMS system





1. Define currents

$\overline{u}(p_4)[-iQ_q e\gamma^\nu]u(p_2)$ quark current

$\overline{u}(p_3)[ie\gamma^\mu]u(p_1)$ electron current

Neglect masses

2. Write $\mathcal{M}$

$$\mathcal{M}_{fi} = \frac{Q_q e^2}{q^2}\left[\overline{u}(p_3)\gamma^\mu u(p_1)\right] g_{\mu\nu} \left[\overline{u}(p_4)\gamma^\nu u(p_2)\right]$$

$$s = p_1 + p_2,\ t = p_1 - p_3 \text{ and } u = p_1 - p_4.$$

Result is:

$$\langle|\mathcal{M}_{fi}|^2\rangle = 2Q_q^2 e^4 \left(\frac{s^2+u^2}{t^2}\right) = 2Q_q^2 e^4 \frac{(p_1.p_2)^2 + (p_1.p_4)^2}{(p_1.p_3)^2},$$

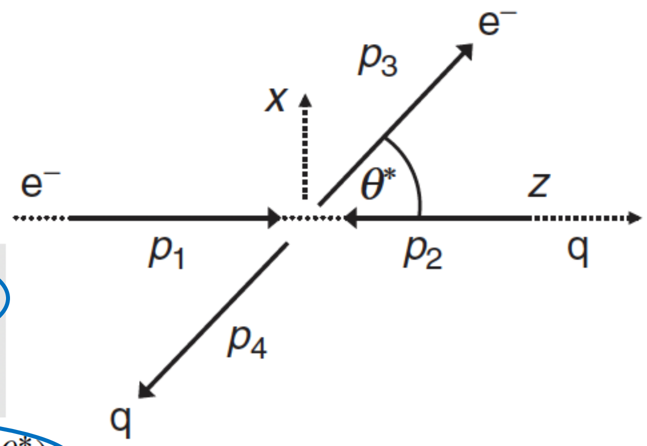
In the CMS system →

$$energy_{CMS} = \sqrt{s}$$

$$define\ E = \frac{\sqrt{s}}{2}$$

→ $-E_q^* = E_e^* = E$ (in the relativistic region $E = \mathrm{p}$)

$$p_1 = (E, 0, 0, +E), \quad p_3 = (E, +E\sin\theta^*, 0, +E\cos\theta^*),$$
$$p_2 = (E, 0, 0, -E), \quad p_4 = (E, -E\sin\theta^*, 0, -E\cos\theta^*).$$



$$\langle|\mathcal{M}_{fi}|^2\rangle = 2Q_q^2 e^4 \left(\frac{s^2+u^2}{t^2}\right) = 2Q_q^2 e^4 \frac{(p_1.p_2)^2 + (p_1.p_4)^2}{(p_1.p_3)^2},$$

$$p_1 \cdot p_2 = 2E^2, \quad p_1 \cdot p_3 = E^2(1-\cos\theta^*) \quad \text{and} \quad p_1 \cdot p_4 = E^2(1+\cos\theta^*).$$

$$\langle|M_{fi}|^2\rangle = 2Q_q^2 e^4 \frac{4E^4 + E^4(1+\cos\theta^*)^2}{E^4(1-\cos\theta^*)^2}$$

Spin averaged !

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s}\frac{\mathrm{p}_f^*}{\mathrm{p}_i^*}|\mathcal{M}_{fi}|^2,$$

$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s}\frac{\left[1 + \frac{1}{4}(1+\cos\theta^*)^2\right]}{(1-\cos\theta^*)^2}$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right].$$

Spin Allowed States in $eq \to eq$

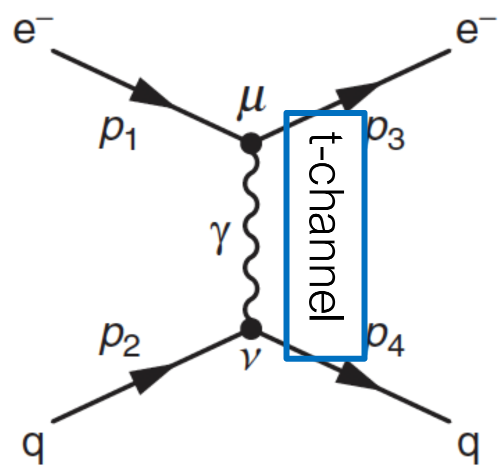
Do we understand these angular terms?

$$\frac{d\sigma}{d\Omega^*} = \frac{Q_q^2 e^4}{8\pi^2 s} \frac{\left[1 + \frac{1}{4}(1+\cos\theta^*)^2\right]}{(1-\cos\theta^*)^2}$$

Helicity is conserved in high energy QED interactions → helicities of quark & electron do not change

Propagator effect

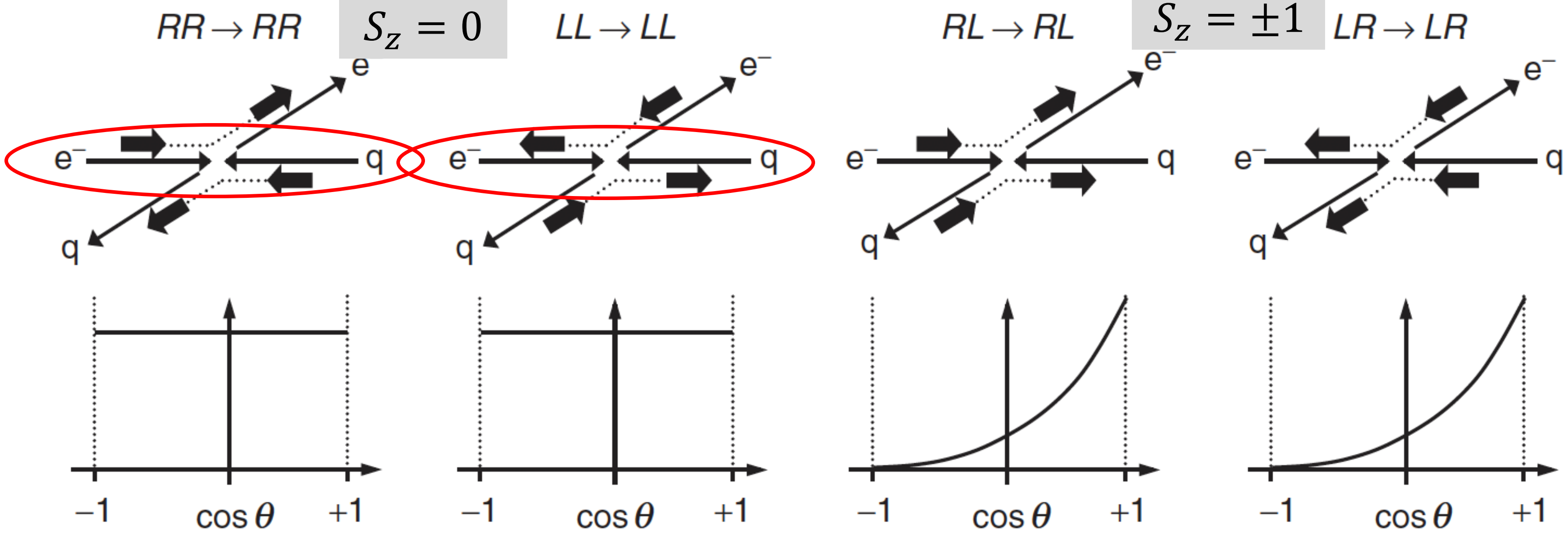
$$q^2 = t = (p_1 - p_3)^2 \approx -E^2(1-\cos\theta^*)$$



The only terms that are non-zero:

$S_z = 0$

$S_z = \pm 1$



→ no preferred direction

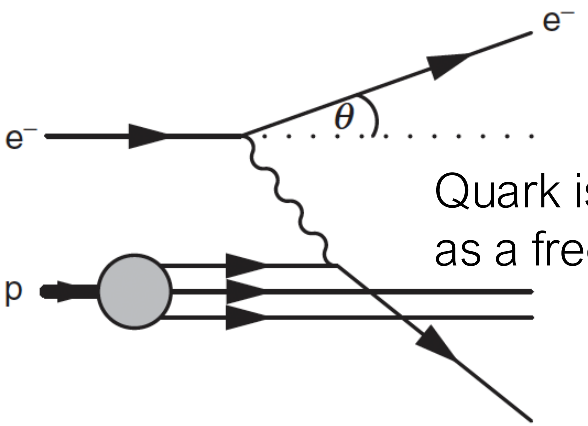
Angular distribution $^1/_4\,(cos\theta^*)^2$

The Quark-Parton Model

$p_1 = (E_1, 0,0, E_1)$ incoming electron
$p_2 = (m_p, 0,0,0)$ proton at rest
$p_3 = (E_3, E_3 \cdot \sin(\theta), 0, E_3 \cdot \cos(\theta))$
$q = (E_1 - E_3, \boldsymbol{p_1} - \boldsymbol{p_3})$

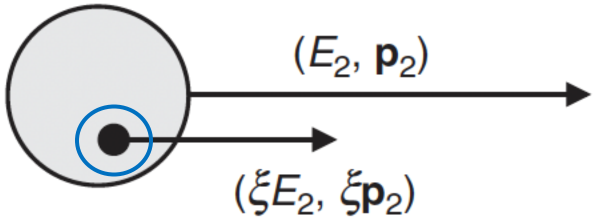
Vocabulary: the proton is made of point-like components termed *partons*

DIS studied in a fast moving frame such that masses can be neglected (“*Infinite momentum frame*”)

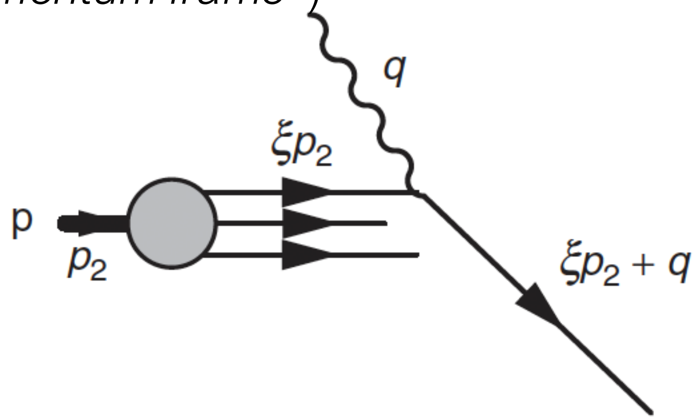


Quark is treated as a free particle!

Proton 4-momentum: $(E_2, \boldsymbol{p_2})$



$$p_q = \xi p_2 = (\xi E_2, 0, 0, \xi E_2)$$



One ‘component’ (quark) inside the proton; carries a fraction ξ of the proton energy E_2

$$\boldsymbol{x = \frac{Q^2}{2p_2 \cdot q}}$$

After the interaction the quark

$$\xi p_2 \to \xi p_2 + q$$

(remember $p^2 = m^2$)

$$(\xi p_2 + q)^2 = \xi^2 p_2^2 + 2\xi p_2 \cdot q + q^2 = m_q^2.$$

$$\xi = \frac{-q^2}{2p_2 \cdot q} = \frac{Q^2}{2p_2 \cdot q} \equiv x.$$

$$(\xi p_2)^2 = m_q^2$$

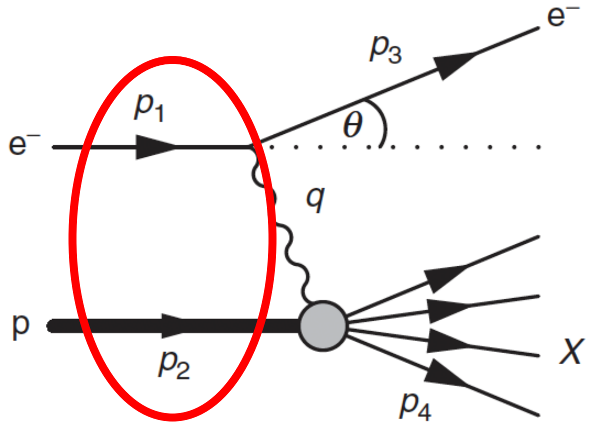
$$(\xi p_2 + q)^2 = m_q^2$$

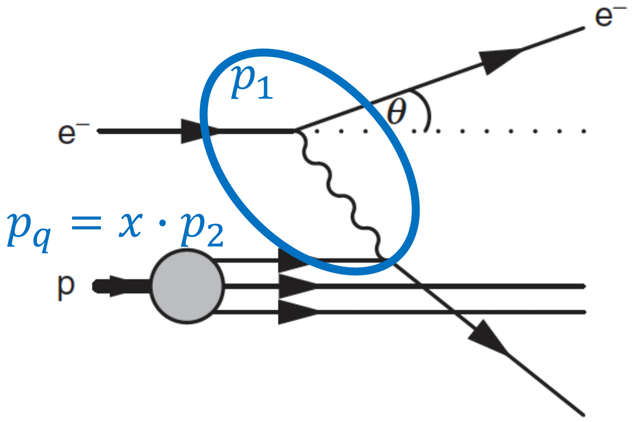
$$\to 2\xi p_2 \cdot q + q^2 = 0$$

$\boldsymbol{x}$ is the fraction of p_2 carried by the quark

ep scattering vs *eq* scattering

Question: how to correlate ep scattering to eq scattering?
Study DIS variables.





Centre Of Mass energy s of the ep system:

$$s = (p_1 + p_2)^2 \approx 2p_1 \cdot p_2$$

Centre Of Mass energy s_q of the eq system:

$$s_q = (p_1 + x \cdot p_2)^2 \approx 2p_1 \cdot x \cdot p_2 = x \cdot s$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s}\right)^2\right].$$

Compute x and y in the ep system:

$$y = p_2 \cdot q / p_2 \cdot p_1$$
$$x = Q^2 / 2 \cdot p_2 \cdot q$$

and in the eq system:

$$y_q = p_q \cdot q / p_q \cdot p_1 = p_2 \cdot x \cdot q / p_2 \cdot x \cdot p_1 = y$$
$$x_q = x = 1 \text{ (elastic scattering)}$$

Rewritten in terms of q^2 and s

eq elastic scattering

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

Elaborate on eq scattering

eq **elastic scattering**

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + \left(1 + \frac{q^2}{s_q}\right)^2\right]$$

$$Q^2 = (s - m_p^2)xy.$$

$$q^2 = -Q^2 = -(s_q - m_q^2)x_q y_q$$

Neglect m_q

$y_q = y$

$x_q = x = 1$

$$\frac{q^2}{s_q} = -x_q y_q = -y.$$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 Q_q^2}{q^4}\left[1 + (1 - y)^2\right]$$

Remember $Q^2 = -q^2$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 Q_q^2}{Q^4}\left[(1 - y) + \frac{y^2}{2}\right],$$

$$\frac{d^2\sigma}{dx\,dQ^2} \approx \frac{4\pi\alpha^2}{Q^4}\left[(1 - y)\frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2)\right]$$

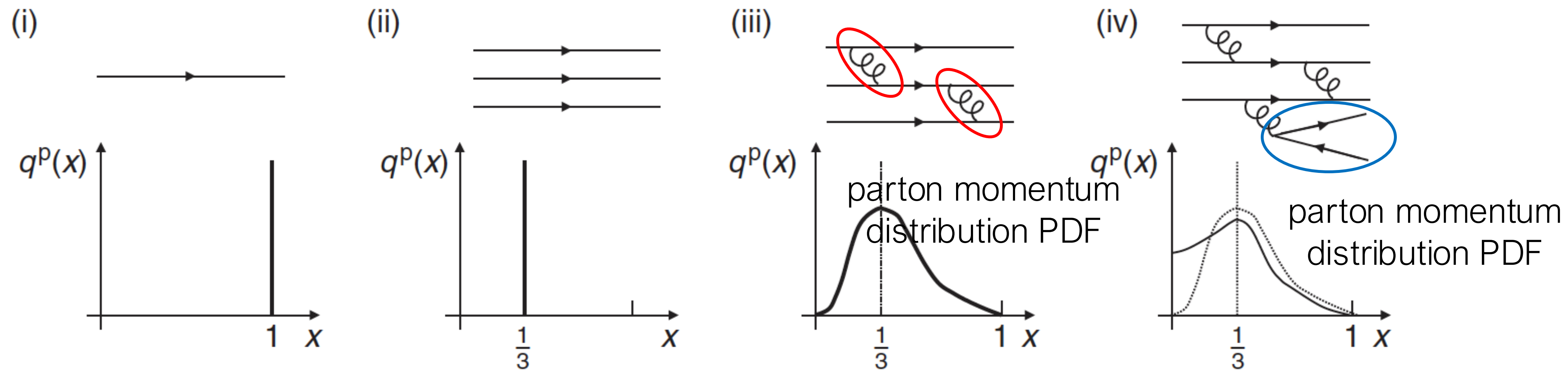
Parton Distribution Functions

Quarks interact inside the proton by exchanging *gluons*

- Exchange of momentum → distribution of x is smeared
- gluons radiate $q\bar{q}$ pairs (mostly low momentum, propagator $\approx \frac{1}{q^2}$)

$u^p(x)\delta x$ number of up-quarks inside the proton between x and x + δx

PDFs are not a priori known and have to be obtained from experiment.



a single point-like particle

three static quarks each 1/3 of p_{proton}

three interacting quarks which can exchange momentum

interacting quarks including higher-order diagrams

$q_i^p(x)$ is PDF

$q_i^p(x)$: *how many quarks of type i&x*

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\times Q_i^2 q_i^p(x)\delta x$$

Quark i

- charge Q_i and
- # quarks i with momentum fraction $q_i^p(x), \cdot x \cdot \delta x$

- Divide by δx
- Sum over all quark flavours → ep scattering double differential cross section

$$\frac{d^2\sigma^{ep}}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)+\frac{y^2}{2}\right]\sum_i Q_i^2 q_i^p(x).$$

Compare with expression containing F_1^{ep} and F_2^{ep}

$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[(1-y)\frac{F_2^{ep}(x,Q^2)}{x}+y^2F_1^{ep}(x,Q^2)\right]$$

$$F_2^{ep}(x,Q^2) = 2xF_1^{ep}(x,Q^2) = x\sum_i Q_i^2 q_i^p(x).$$

The parton model:

- underlying process is elastic point-like objects: eq → no (strong) Q^2 dependence expected;
- $F1$ and F_2 can be written as functions of x alone: $F_1(x, Q^2) \rightarrow F_1(x)$ and $F_2(x, Q^2) \rightarrow F_2(x)$.
- $F_1(x) = 2xF_2(x)$: elastic underlying process scattering between spin 1/2 Dirac particles; the quark magnetic moment fixes structure functions are fixed with respect to one another.

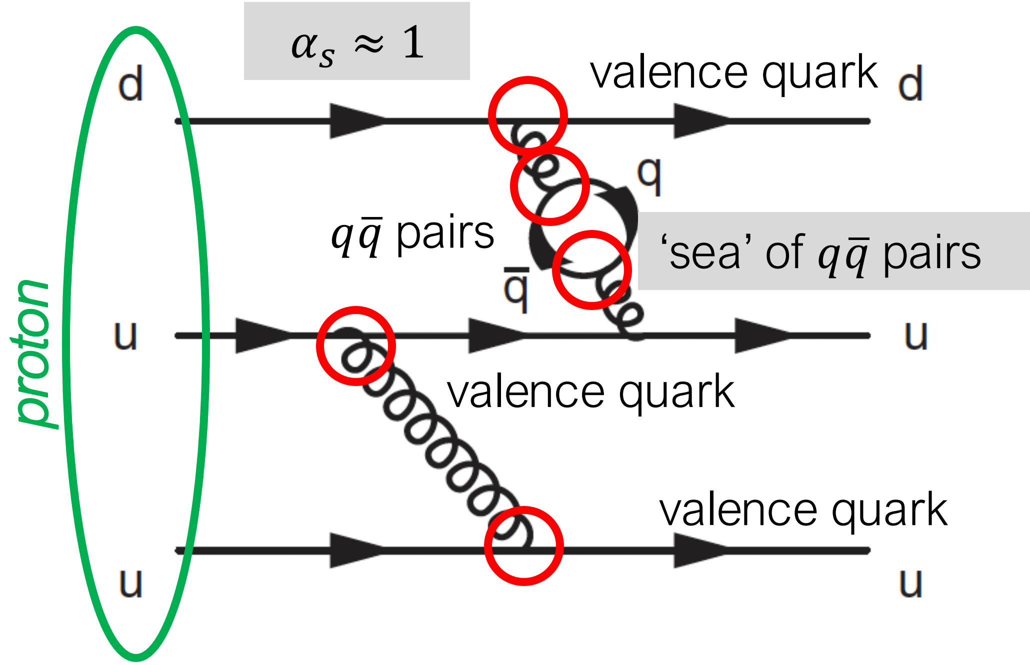
Determining Parton Structure Functions

How to determine Parton Structure Functions?

Problems:

- $\alpha_s \approx 1$ → cannot do a perturbation expansion of the pp scattering (coupling constant has to be small);
- The proton consists of components bound together → components interact with each other;
- Gluons couple to quarks → $q\bar{q}$ pairs are created/annihilated

→ Structure Functions have to be measured, cannot get them from *'first principles'*



- $q\bar{q}$ pairs are mostly created at low x values due to $1/q^2$ gluon propagator;
- ep inelastic scattering due to quarks and antiquarks;
- Quarks of all types may contribute but relevance decreases with increasing quark masses → $s\bar{s}$ can already be neglected (small contribution, even smaller from heavier quarks).

Determining Parton Structure Functions - 2

Write explicitly:

$$F_2^{ep}(x, Q^2) = 2xF_1^{ep}(x, Q^2) = x\sum_i Q_i^2 q_i^p(x).$$

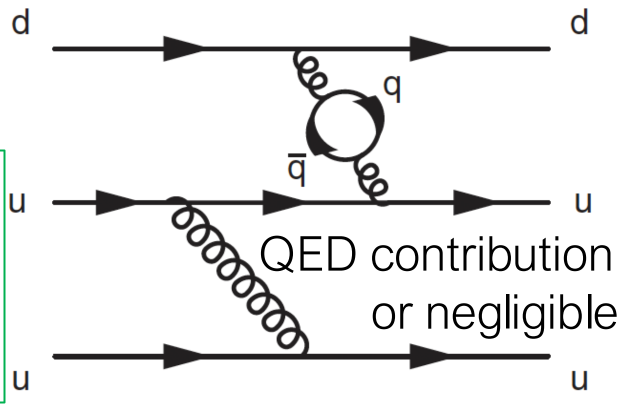
- In principle $q_i^p \neq q_i^n$
- $u^p(x)$ x-distribution of 'up' quarks in the proton (shape + integral)
- $u^p(x)$ = valence quarks + sea quarks
- $\overline{u^p}(x)$ only sea quarks

Charge[2]

$$F_2^{ep}(x) = x\sum_i Q_i^2 q_i^p(x) \approx x\left(\frac{4}{9}u^p(x) + \frac{1}{9}d^p(x) + \frac{4}{9}\overline{u}^p(x) + \frac{1}{9}\overline{d}^p(x)\right)$$

$$F_2^{en}(x) = x\sum_i Q_i^2 q_i^n(x) \approx x\left(\frac{4}{9}u^n(x) + \frac{1}{9}d^n(x) + \frac{4}{9}\overline{u}^n(x) + \frac{1}{9}\overline{d}^n(x)\right)$$

Protons and neutrons have very similar characteristics (isospin symmetry)
→ can exchange up and down



QED contribution small or negligible

Not only gluons, also photons but $\alpha_s \approx 1 \gg \alpha_{EM}$

$$d^n(x) = u^p(x) \equiv u(x) \quad \text{and} \quad u^n(x) = d^p(x) \equiv d(x)$$

$$\overline{d}^n(x) = \overline{u}^p(x) \equiv \overline{u}(x) \quad \text{and} \quad \overline{u}^n(x) = \overline{d}^p(x) \equiv \overline{d}(x)$$

Determining Parton Structure Functions - 3

$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)$$

$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)$$

$u^n(x) \rightarrow d^p(x)$

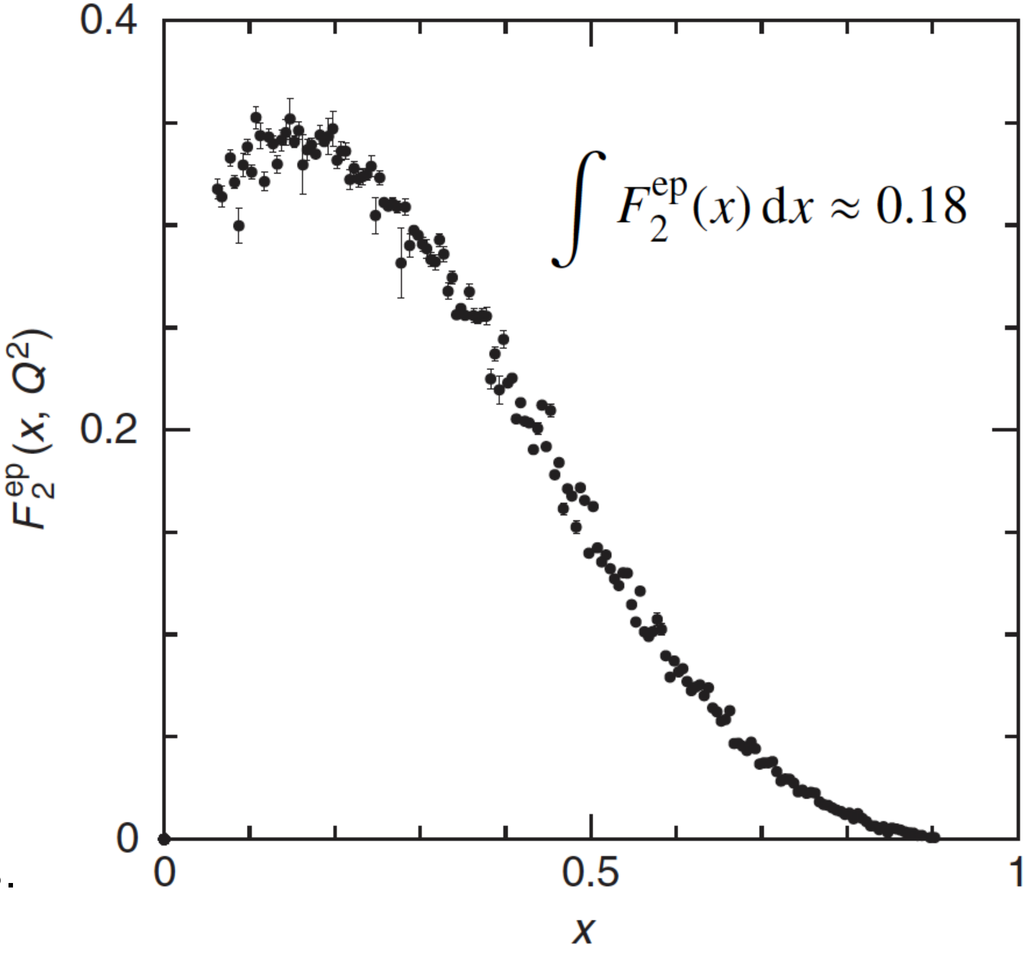
Integral, not shape

$$\int_0^1 F_2^{ep}(x)\,\mathrm{d}x = \frac{4}{9}f_u + \frac{1}{9}f_d \quad \text{and} \quad \int_0^1 F_2^{en}(x)\,\mathrm{d}x = \frac{4}{9}f_d + \frac{1}{9}f_u$$

charge[2] charge[2]

$$f_u = \int_0^1 [xu(x) + x\overline{u}(x)]\,\mathrm{d}x \quad \text{and} \quad f_d = \int_0^1 [xd(x) + x\overline{d}(x)]\,\mathrm{d}x$$

f_u fraction of proton momentum carried by up- and anti-up quarks.
f_d fraction carried by the down-/anti-down quarks.



f_u and f_d obtained directly from the experimental measurements of the proton and neutron structure functions.

Integral of F_2^{en} and F_2^{ep}

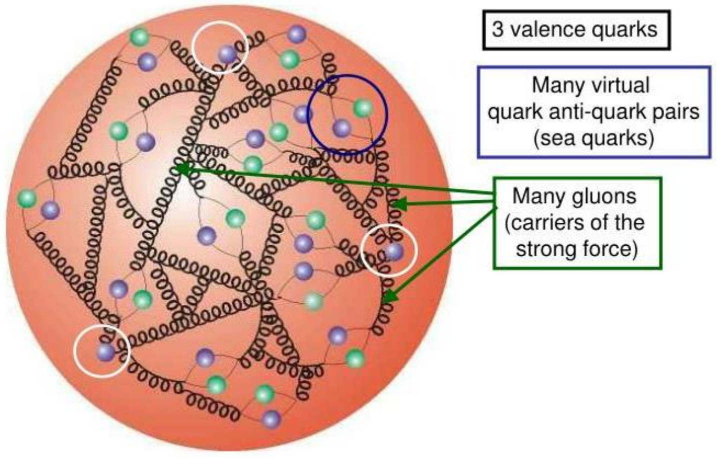
$$\int F_2^{ep}(x)\,dx \approx 0.18 \qquad \int F_2^{en}(x)\,dx \approx 0.12$$

$$\int_0^1 F_2^{ep}(x)\,dx = \frac{4}{9}f_u + \frac{1}{9}f_d \quad \text{and} \quad \int_0^1 F_2^{en}(x)\,dx = \frac{4}{9}f_d + \frac{1}{9}f_u$$

Using the relation above you get $f_u \approx 0.36$ and $f_d \approx 0.18$ $\quad f_u/f_d \approx 2/1$

proton = two up-quarks and one down-quark not surprising that $f_u/f_d \approx 2/1$

- ***More important***: the total fraction of the momentum of the proton carried by quarks and antiquarks is ~ 50%;
- the rest is carried by the gluons, mediators of the strong interaction;
- Because the gluons are electrically neutral, they do not contribute to the QED process of electron–proton deep inelastic scattering.



Proton very complex structure.

Gluons → $q\bar{q}$ pairs generated/absorbed → two big types of quarks:

- Valence quarks, determine quantum numbers of the proton;
- Sea quarks, produced in pairs

→

$u(x)$ & $d(x)$ [....] are made of valence and sea quarks at the same time;

$$u(x) = u_V(x) + u_S(x) \quad \text{and} \quad d(x) = d_V(x) + d_S(x).$$

$\bar{u}(x)$ & $\bar{d}(x)$ [....] are made of sea quarks only;

$$\bar{u}(x) \equiv \bar{u}_S(x) \quad \text{and} \quad \bar{d}(x) \equiv \bar{d}_S(x)$$

How many $u(x)$ & $d(x)$ quarks inside the proton (uud)?

$$\int_0^1 u_V(x)\,dx = 2 \quad \text{and} \quad \int_0^1 d_V(x)\,dx = 1$$

How many $\bar{u}(x)$ & $\bar{d}(x)$ quarks inside the proton (made of valence quarks only uud)?
Since quarks and antiquarks are produced in pairs and since $m_u \approx m_d$ → equally populated of $u_s, d_s, \bar{u}_s, \bar{d}_s$

$$u_S(x) = \bar{u}_S(x) \approx d_S(x) = \bar{d}_S(x) \approx S(x)$$

Qualitative Arguments F_2^{ep}

If we start from the expression for F_2 derived before

Callan-Gross relation

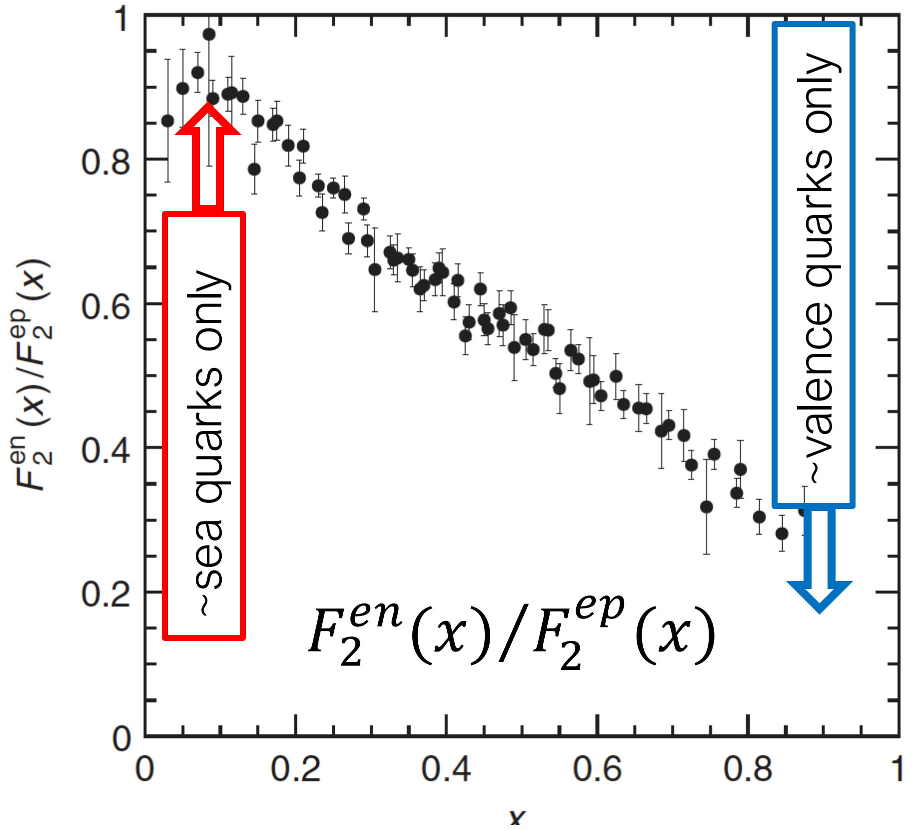
$$F_2^{ep}(x) = 2xF_1^{ep}(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x) + \frac{4}{9}\overline{u}(x) + \frac{1}{9}\overline{d}(x)\right)$$

$$F_2^{en}(x) = 2xF_1^{en}(x) = x\left(\frac{4}{9}d(x) + \frac{1}{9}u(x) + \frac{4}{9}\overline{d}(x) + \frac{1}{9}\overline{u}(x)\right)$$

$$F_2^{ep}(x) = x\left(\frac{4}{9}u_V(x) + \frac{1}{9}d_V(x) + \frac{10}{9}S(x)\right)$$

$$F_2^{en}(x) = x\left(\frac{4}{9}d_V(x) + \frac{1}{9}u_V(x) + \frac{10}{9}S(x)\right)$$

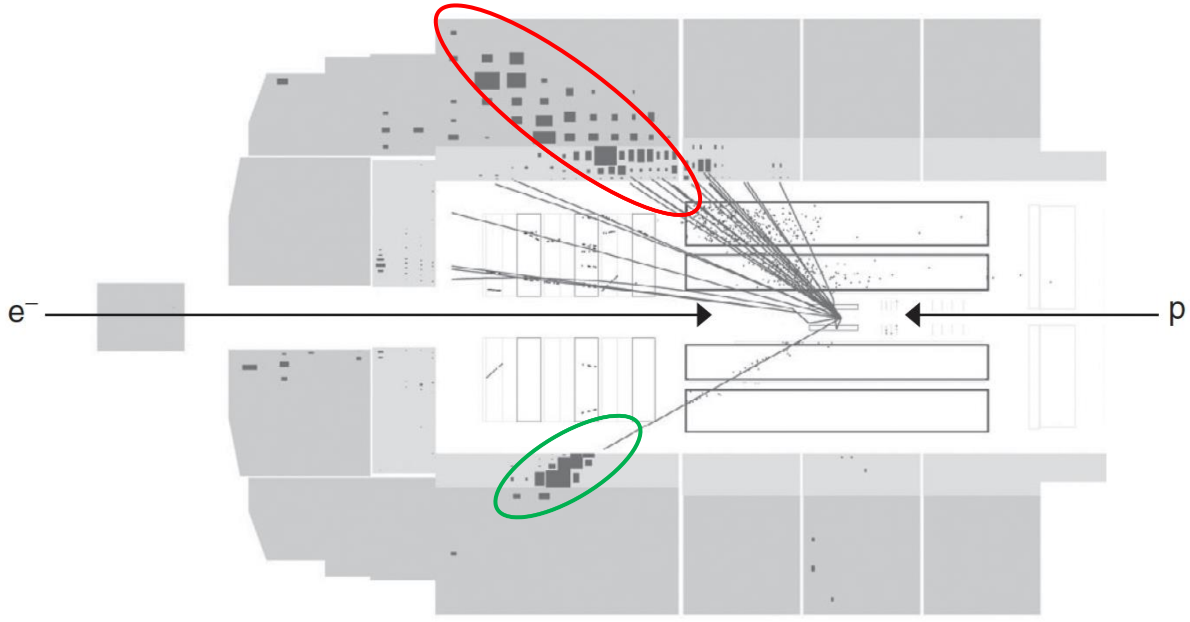
$$\frac{F_2^{en}(x)}{F_2^{ep}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)}$$

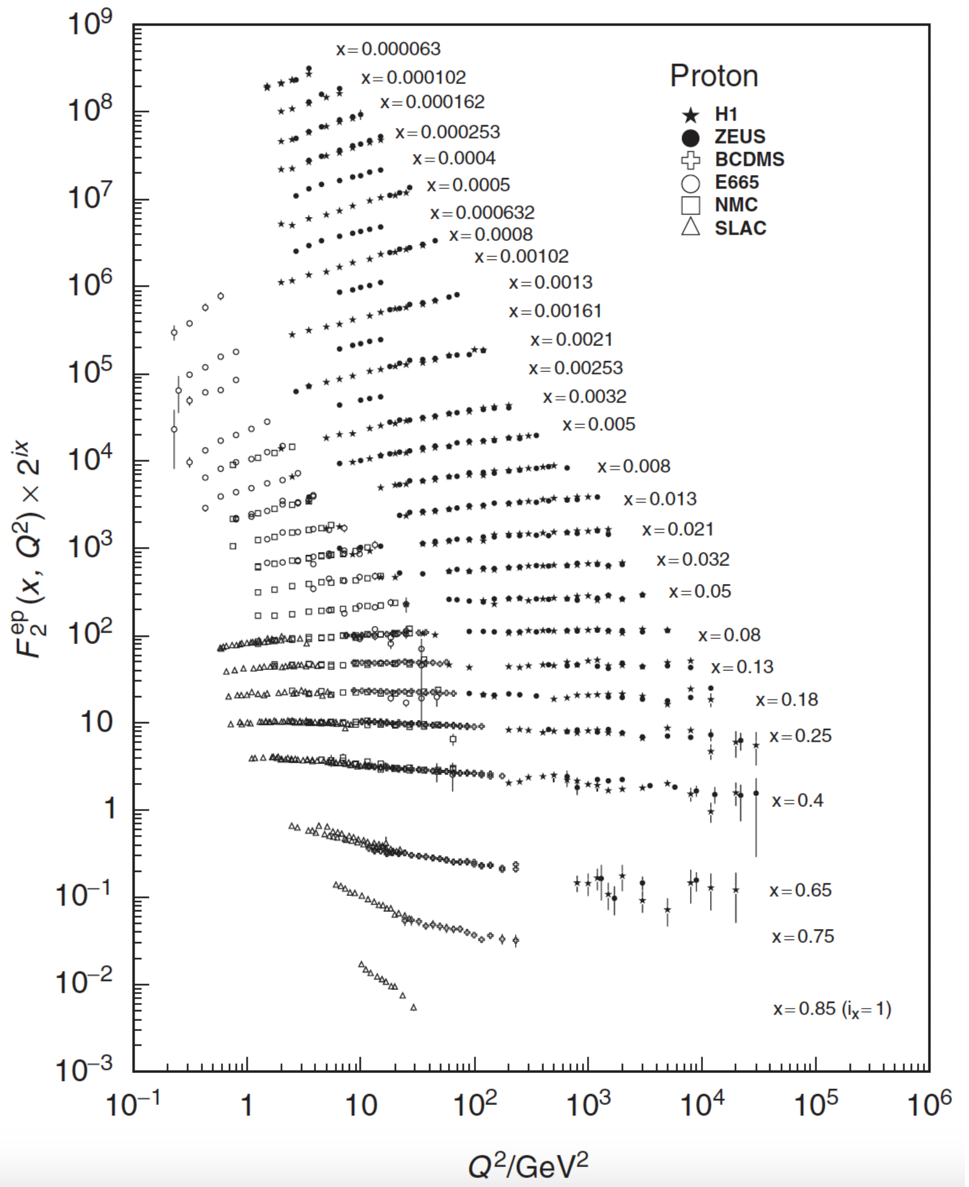


Qualitative arguments:

- Sea quarks produced at low x → dominant at $x\sim 0$ $\quad \frac{F_2^{en}(x)}{F_2^{ep}(x)} \to 1$ as $x \to 0$. OK!
- Valence quarks mostly at $x\sim 1$ *(expect 2/3)* $\quad \frac{F_2^{en}(x)}{F_2^{ep}(x)} \to \frac{1}{4}$ as $x \to 1$. ?

Structure Functions at Hera

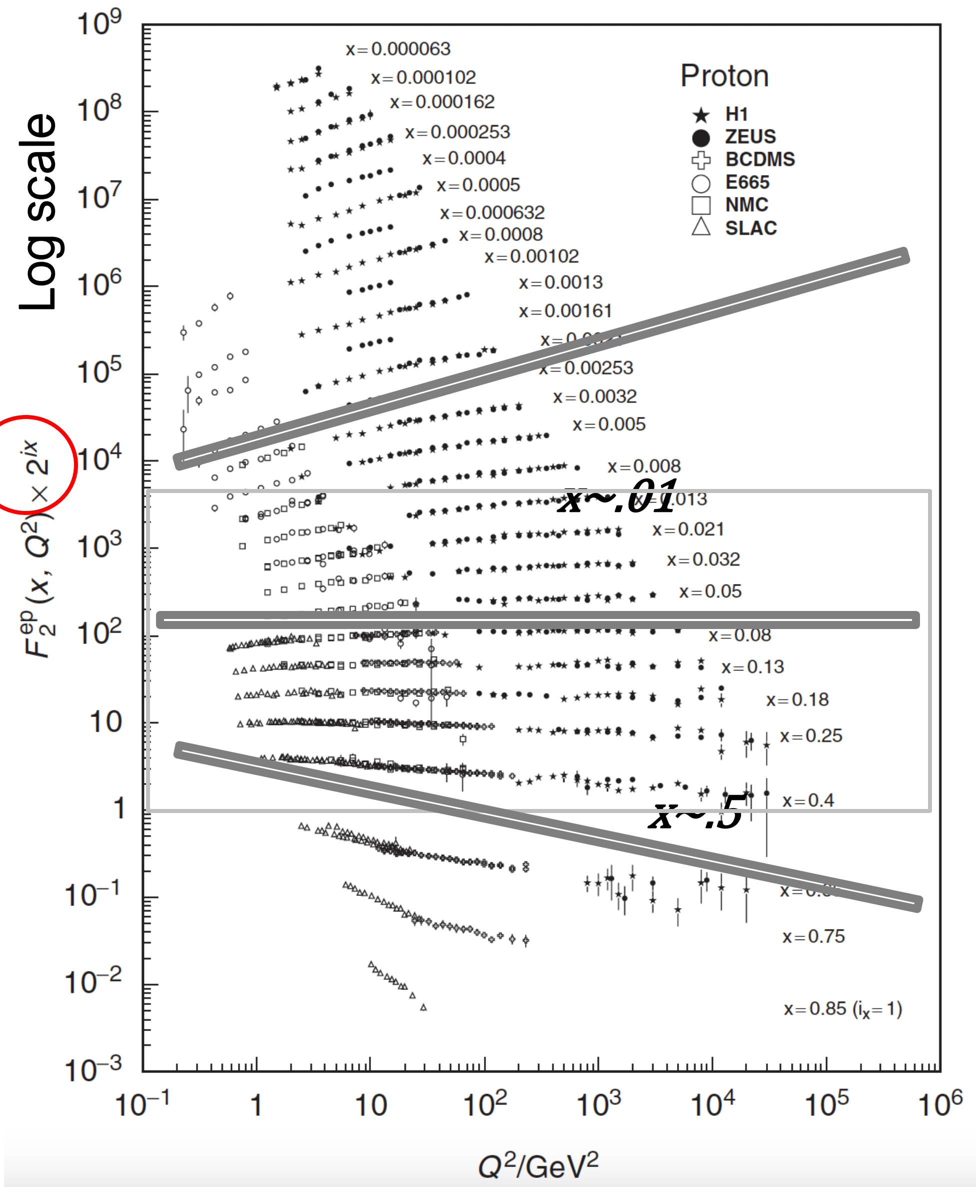




- The final-state hadronic system: a jet of high-energy particles. The energy and direction of this jet of particles is measured badly than
- electron showers;

→ Q^2 and x are determined from the energy and scattering angle of the electron.

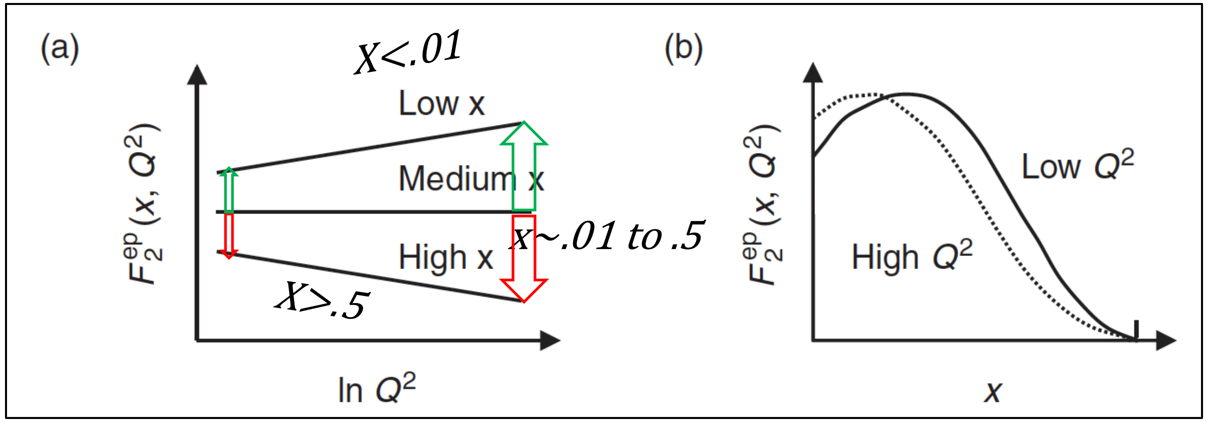
Scaling Violations



We observed that $F_1(x, Q^2) \rightarrow F_1(x)$ and $F_2(x, Q^2) \rightarrow F_2(x)$.

OK between *x~.01 to .5* → quark point-like object (small dep. on Q^2)

Outside this interval, data show '*scaling violations*'



high Q^2 the measured structure functions → lower values of x relative to the structure functions at low Q^2

scaling violation

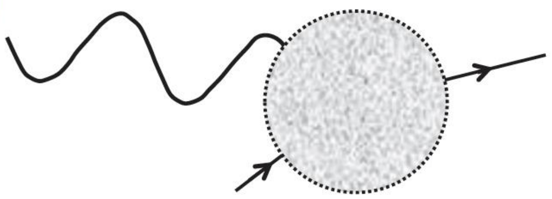
→ at high Q^2, the proton is observed to have a greater fraction of low x quarks

Scaling Violations

Do we understand why scaling violations?

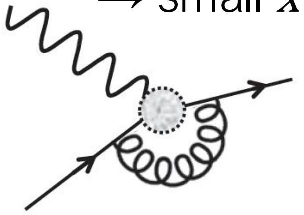
Low Q^2 photon
→ sees 'little'

(a)

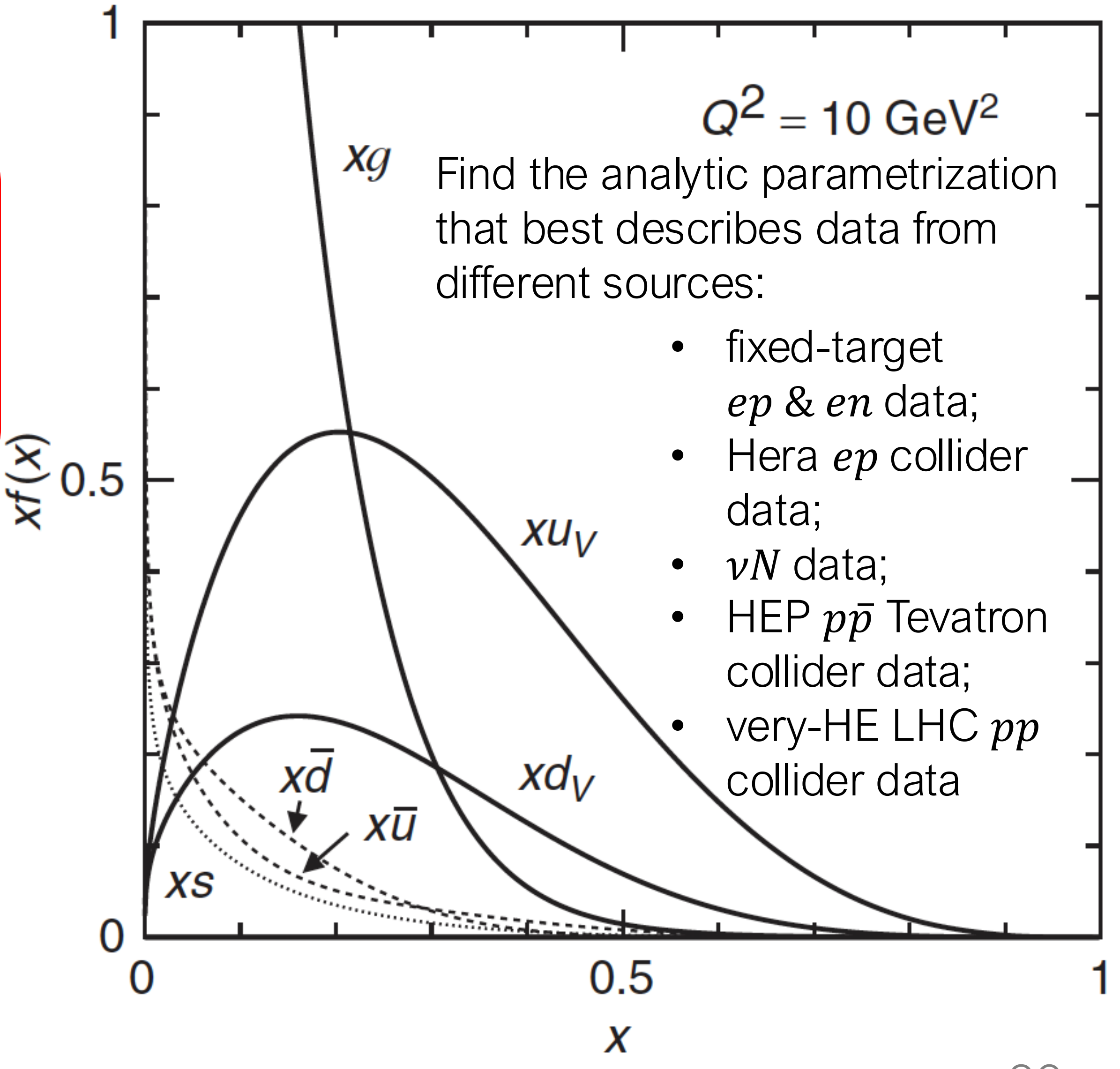


High Q^2 photon
→ sees '$q \to q + g$
→ small x

(b)



- proton PDFs cannot be calculated from first principles;
- the Q^2 dependence of the PDFs is calculable DGLAP (Dokshitzer–Gribov–Lipatov–Altarelli–Parisi) equations (QCD).
- These equations are based on parton splitting functions for the QCD processes $q \to qg \;\; and \;\; g \to q\bar{q}$



Find the analytic parametrization that best describes data from different sources:

- fixed-target ep & en data;
- Hera ep collider data;
- νN data;
- HEP $p\bar{p}$ Tevatron collider data;
- very-HE LHC pp collider data

Scaling Violations in DIS

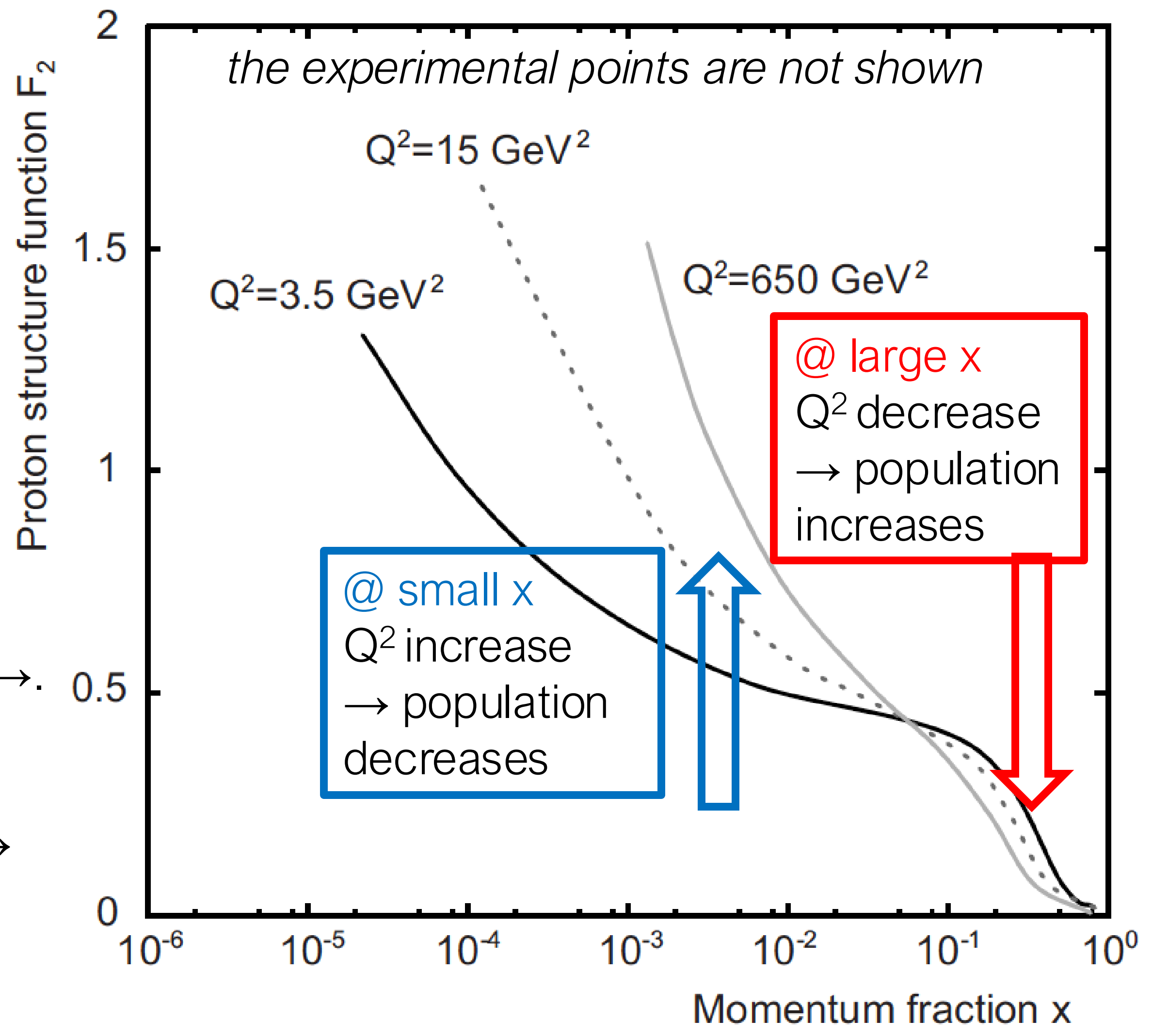
Quarks can emit or absorb gluons, gluons may split into $q\bar{q}$ pairs, or emit gluons themselves. Thus, the momentum distribution between the constituents of the nucleon is changing continuously.

We see that the structure function

- increases with Q^2 at small values of x and
- decreases when Q^2 increases at large values of x .

This behaviour, called **scaling violation**, is sketched in Fig. →.

With increasing values of Q^2 many quarks seen → the momentum of the proton is shared among many partons → there are few quarks with large momentum fractions in the nucleon → quarks with small momentum fractions predominate.



Extrapolating Structure Functions

The number of partons seen to share the momentum of the nucleon increases when Q^2 increases.

Problem!

How to extrapolate measured $F_2(x)$ to higher values of Q^2?
How do we go Hera → LHC

The dependence of the quark and gluon distributions can be described by a system of coupled integral-differential equations [Altarelli Parisi equations].

- If $\alpha_s(Q^2)$ and the shape of $q(x,Q^2)$ and $g(x,Q^2)$ are known at a given value Q^2
- → $q(x,Q^2)$ and $g(x,Q^2)$ can be predicted from QCD for all other values of Q^2.
- The coupling $\alpha_s(Q^2)$ and the gluon distribution $g(x,Q^2)$, which cannot be directly measured, can be determined from the observed scaling violation of the structure function $F_2(x,Q^2)$.

Altarelli – Parisi Equations (Review Particles Properties)

In QCD, the above process is described in terms of scale-dependent parton distributions $f_a(x,\mu^2)$, where $a = g$ or q and, typically, μ is the scale of the probe Q. For $Q^2 \gg M^2$, the structure functions are of the form

$$F_i = \sum_a C_i^a \otimes f_a, \tag{16.21}$$

where $\otimes$ denotes the convolution integral

$$C \otimes f = \int_x^1 \frac{dy}{y} C(y)\, f\left(\frac{x}{y}\right), \tag{16.22}$$

and where the coefficient functions C_i^a are given as a power series in α_s. The parton distribution f_a corresponds, at a given x, to the density of parton a in the proton integrated over transverse momentum k_t up to μ. Its evolution in μ is described in QCD by a DGLAP equation (see Refs. 14–17) which has the schematic form

$$\frac{\partial f_a}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} \sum_b (P_{ab} \otimes f_b), \tag{16.23}$$

where the P_{ab}, which describe the parton splitting $b \to a$, are also given as a power series in α_s. Although perturbative QCD can predict, via Eq. (16.23), the evolution of the parton distribution functions from a particular scale, μ_0, these DGLAP equations cannot predict them *a priori* at any particular μ_0. Thus they must be measured at a starting point μ_0 before the predictions of QCD can be compared to the data at other scales, μ. In general, all observables involving a hard hadronic interaction (such as structure functions) can be expressed as a convolution of calculable, process-dependent coefficient functions and these universal parton distributions, e.g. Eq. (16.21).

It is often convenient to write the evolution equations in terms of the gluon, non-singlet (q^{NS}) and singlet (q^S) quark distributions, such that

$$q^{NS} = q_i - \bar{q}_i \quad (\text{or } q_i - q_j), \qquad q^S = \sum_i (q_i + \bar{q}_i). \tag{16.24}$$

The non-singlet distributions have non-zero values of flavor quantum numbers, such as isospin and baryon number. The DGLAP evolution equations then take the form

$$\frac{\partial q^{NS}}{\partial \ln \mu^2} = \frac{\alpha_s(\mu^2)}{2\pi} P_{qq} \otimes q^{NS},$$

$$\frac{\partial}{\partial \ln \mu^2} \begin{pmatrix} q^S \\ g \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi} \begin{pmatrix} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} q^S \\ g \end{pmatrix}, \tag{16.25}$$

where P are splitting functions that describe the probability of a given parton splitting into two others, and n_f is the number of (active) quark flavors. The leading-order

Nomenclature

$f_a(x,q^2)$ parton distributions
P_{ab} parton splitting → ab
n_f number of active quark flavors

Altarelli – Parisi Equations (Review Particles Properties)

into two others, and n_f is the number of (active) quark flavors. The leading-order Altarelli-Parisi [16] splitting functions are

$$P_{qq} = \frac{4}{3}\left[\frac{1+x^2}{(1-x)}\right]_+ = \frac{4}{3}\left[\frac{1+x^2}{(1-x)_+}\right] + 2\delta(1-x)\ , \quad (16.26)$$

$$P_{qg} = \frac{1}{2}\left[x^2 + (1-x)^2\right]\ , \quad (16.27)$$

$$P_{gq} = \frac{4}{3}\left[\frac{1+(1-x)^2}{x}\right]\ , \quad (16.28)$$

$$P_{gg} = 6\left[\frac{1-x}{x} + x(1-x) + \frac{x}{(1-x)_+}\right] + \left[\frac{11}{2} - \frac{n_f}{3}\right]\delta(1-x), \quad (16.29)$$

where the notation $[F(x)]_+$ defines a distribution such that for any sufficiently regular test function, $f(x)$,

$$\int_0^1 dx f(x)[F(x)]_+ = \int_0^1 dx\ (f(x) - f(1))F(x)\ . \quad (16.30)$$

In general, the splitting functions can be expressed as a power series in α_s. The series contains both terms proportional to $\ln\mu^2$ and to $\ln\ 1/x$. The leading-order DGLAP evolution sums up the $(\alpha_s \ln\mu^2)^n$ contributions, while at next-to-leading order (NLO) the sum over the $\alpha_s(\alpha_s \ln\mu^2)^{n-1}$ terms is included [18,19]. In fact, the NNLO contributions to the splitting functions and the DIS coefficient functions are now also all known [20–22].

In the kinematic region of very small x, it is essential to sum leading terms in $\ln\ 1/x$, independent of the value of $\ln\mu^2$. At leading order, LLx, this is done by the BFKL equation for the unintegrated distributions (see Refs. [23,24]). The leading-order $(\alpha_s \ln(1/x))^n$ terms result in a power-like growth, $x^{-\omega}$ with $\omega = (12\alpha_s \ln 2)/\pi$, at asymptotic values of $\ln\ 1/x$. More recently, the next-to-leading $\ln\ 1/x$ (NLLx) contributions have become available [25,26]. They are so large (and negative) that the result appears to be perturbatively unstable. Methods, based on a combination of collinear and small x resummations, have been developed which reorganize the perturbative series into a more stable hierarchy [27–30]. There are indications that small x resummations become necessary for real precision for $x \lesssim 10^{-3}$ at low scales. On the

Symmetries / Significant properties

- P_{qg}, P_{gg}: symmetric $x \leftrightarrow (1-x)$
- P_{qq}, P_{gg}: diverge for $x \to 1$
- P_{gq}, P_{gg}: diverge for $x \to 0$
- $P_{qq'} = 0$
- $P_{\bar{q}g} = P_{qg}$